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Quantum Logical Bioinformatics: The Genetic Alphabet of 4 Unitary Hadamard Operators and Cyclic Groups

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This paper presents data on an additional operator-alphabetic informatic system for biological inheritance in living organisms, which can provide inheritance of biological algorithms and functions and which exists alongside the well-known nucleotide-alphabetic system of information inheritance. This operator-alphabetic bioinformatics and its alphabet are linked to quantum mechanics, quantum logic, and quantum information science, since genetic molecules belong to the microworld of quantum mechanics. In the search for and justification of this operator bioinformatics and its alphabets, special attention is paid to unitary operators, which underlie all calculations in quantum computers and which, in quantum mechanics, describe the evolution of closed quantum systems. The quantum logic apparat being developed for this bioinformatics is based on a genetic alphabet of four unitary Hadamard matrices, families of unitary matrices based on this alphabet, and cyclic groups of unitary transformations. Some prospects of the proposed approach for the development of quantum logic biology, artificial intelligence, and biotechnology are discussed.

Keywords: *quantum logic, quantum bioinformatics, genetic automata, cycles, biorhythms, probability, unitary matrices, tensor product, systolic processors, artificial intelligence, qubits.*

1. Introduction

Information communication systems are built on the use of appropriate alphabets to form information messages. For example, all computer programs rely on corresponding alphabets. The amino acid sequences of proteins are genetically inherited using information messages in DNA and RNA molecules based on an alphabet of four DNA and RNA nucleotides: adenine A, guanine G, cytosine C, and thymine T (in RNA, uracil U replaces thymine T). Knowledge of this alphabet is useful for understanding the structure of proteins and nucleic acids. However, as Nobel laureate in chemistry T. Steitz emphasizes, all knowledge about the biochemical structure of proteins and nucleic acids encoded in the genome will not tell us, for example, how a butterfly flies [23]. Nor will it tell us how turtles, after hatching from an egg, immediately, without any training, begin to crawl toward the water with coordinated movements of their limbs, which requires the logically coordinated activity of millions of their nerve and muscle cells. Or how a newborn baby cries and begins to suckle at its mother's breast, which also requires the logically coordinated activity of billions of nerve and muscle cells. Knowledge of DNA/RNA nucleotide sequences also fails to explain the inheritance of the mathematical beauty of bioforms (mollusk shells, etc.), which are repeated in bodies of very different biochemical compositions and are constructed through the spatiotemporal ordering of trillions of different molecules. Thus, the modern science of biological inheritance lacks knowledge of a bioinformation system capable of ensuring the inheritance of "cooperative biomechanics" phenomena, that is, the logically coordinated (coherent) behavior of body parts and the functions based on them.

The aforementioned inherited logical forms of collective behavior in biosystems require a search for a corresponding operator bioinformatics system based on a suitable alphabet for their modeling. One can believe, that, alongside nucleotide-alphabetic bioinformatics, an additional operator-alphabetic bioinformatics operates in living things. This hidden operator bioinformatics and its alphabet are apparently linked to quantum mechanics and quantum information, since genetic molecules belong to the microworld of quantum mechanics, and many authors have long suspected that living organisms are quantum-like entities. In search of an appropriate alphabet, special attention should be paid to unitary operators, which play a role of logical gates to provide all calculations in quantum computing [15] and which, in quantum mechanics, describe the evolution of closed quantum systems.

This article is devoted to the author's discovery of a genetic alphabet of four Hadamard unitary operators and the development of operator quantum-logical bioinformatics based on it for the mathematical modeling of logical features of the structure and behavior of inherited multicomponent body parts, including the properties of multiple, time-coordinated cyclic processes. The article provides specific examples of modeling known biological phenomena from the perspective of this emerging operator bioinformatics, which relies on the aforementioned alphabet of unitary operators, its algebraic extensions, cyclic groups of unitary operators, quantum logic formalisms, and Chargaff's second rule, well-known in genetics. These examples confirm the adequacy of the emerging quantum-logical bioinformatics system, which offers new approaches to modeling inherited, logically organized biological phenomena.

The founder of quantum information science, Yu. I. Manin (https://en.wikipedia.org/wiki/Yuri_Manin), introduced the concept of a quantum computer in his book [14, p. 15] precisely when analyzing the characteristics of high-speed processing of information in chromosomal DNA by "*genetic automata*," prophetically pointing out the important role of unitary operators and tensor products: "*A quantum automaton must be abstract: its mathematical model must use only the most general quantum principles, without prejudging* *physical implementations. Then the evolution model is a unitary rotation in a finite-dimensional Hilbert space, and the model of virtual separation into subsystems corresponds to the decomposition of space into a tensor product. Somewhere in this picture, there must be a place for interaction, traditionally described by Hermitian operators and probabilities*." Thus, the very birth of quantum information science, so promising for the development of artificial intelligence and information science. The data in this article are consistent with this prophecy of Yu. I. Manin.

2. Genetic Matrices of DNA Nucleotide Alphabets and the Alphabet of 4 Unitary Hadamard Operators

Amino acid sequences of proteins are inherited through information messages on genetic DNA molecules, written in an alphabet of 4 nucleotides: adenine A, cytosine C, guanine G, and thymine T. This alphabet carries a system of binary-opposition indicators, which provides distinguishing three types of binary sub-alphabets within it:

1) two of these nucleotides are purines (A and G), having two rings in their molecule, and the other two (C and T) are pyrimidines, containing one ring. This yields a binary representation (binary sub-alphabet) **C**=**T**=**0**, **A**=**G**=**1**;

2) two of these nucleotides are keto-molecules (T and G), and the other two (C and A) are amino-molecules, yielding the binary representation **C**=**A**=**0**, **T**=**G**=**1**;

3) pairs of complementary nucleotides A-T and C-G are linked by 2 and 3 hydrogen bonds, respectively (called weak and strong hydrogen bonds in genetics), yielding the binary representation **C**=**G**=**0**, **A**=**T**=**1**.

These molecular binary-oppositional traits of the nucleotide alphabet of DNAs (and RNAs) of all living organisms are summarized in Table 1, which shows the distribution of traits within it.

Table 1. Binary distribution of molecular traits in the DNA nucleotide alphabet G, A, T, C. The fourth row of the table represents with the symbol +1 the fact of the presence of benzene rings in the molecules of all four nucleotides.

Molecular traits and their symbols	G	A	T	C
pyrimididines +1, purines -1	-1	-1	+1	+1
amino-nucleotides +1, кето-nucleotides - 1	-1	+1	-1	+1
complementarity with 3 or 2 hydrogen bonds: +1, -1	+1	-1	-1	+1
the presence of benzene rings +1	+1	+1	+1	+1

The right-hand side of this phenomenological Table 1 contains an appeared fourth-order Hadamard matrix, whose quadrants are occupied by four types of second-order Hadamard matrices (unitary when normalized). Hadamard matrices are fundamental building blocks of quantum computers, enabling qubit superposition and also a key element of quantum parallelism in the quantum Fourier transform and other quantum algorithms.

This same set of four Hadamard matrices is revealed in the analysis of molecular genetic informatics from other approaches. For example, in the construction of genetic ($2^n \cdot 2^n$)-matrices, in which the columns are numbered with binary digits of one of the aforementioned DNA sub-alphabets, and the rows with digits of another. In this case, the same four matrices are derived from the 1:3 oppositional relationships between the four members of the DNA nucleotide alphabet (for example, only one nucleotide, thymine T, is phenomenologically replaced in the molecular genetic system by uracil U during the transition from DNA to RNA). In another approach, taking into account the well-known second Chargaff rule [5, 28] on the ratio of probabilities of 4 nucleotides in all long single-stranded DNAs (length greater than 100 kbps) leads to a real Hermitian probability matrix $\mathbf{W_D}$ = [0.5, 0.5; 0.5, 0.5]. This genetic Hermitian matrix $\mathbf{W_D}$ is doubly stochastic: the sum of the elements in each row and each column of such a matrix is equal to one. But with respect to doubly stochastic matrices, the following theorem is known [21]:

- If the matrix $\mathbf{V} = ||\mathbf{v_{ij}}||^n$ is unitary, then the matrix $\mathbf{W} = ||\mathbf{w_{ij}}||^n$, where $\mathbf{w_{ij}} = |\mathbf{v_{ij}}|^2$, is doubly stochastic.

According to this theorem, the doubly stochastic genetic matrix $\mathbf{W_D}$ of probabilities corresponds to four unitary Hadamard genetic matrices, denoted here as $\mathbf{H_C}$, $\mathbf{H_A}$, $\mathbf{H_T}$, and $\mathbf{H_G}$ (Fig. 1): squaring all components of each of these unitary matrices generates the doubly stochastic matrix WD. Note that these same four unitary matrices $\mathbf{H_C}$, $\mathbf{H_A}$, $\mathbf{H_T}$, and $\mathbf{H_G}$, but taken with a minus sign, are treated as their banal analogue and are not considered separately.

$$\mathbf{H_C} = 2^{-0.5}\begin{vmatrix} 1, -1 \\ 1,\ 1 \end{vmatrix}; \quad \mathbf{H_T} = 2^{-0.5}\begin{vmatrix} 1,\ 1 \\ 1, -1 \end{vmatrix}; \quad \mathbf{H_G} = 2^{-0.5}\begin{vmatrix} 1,\ 1 \\ -1,\ 1 \end{vmatrix}; \quad \mathbf{H_A} = 2^{-0.5}\begin{vmatrix} -1,\ 1 \\ 1,\ 1 \end{vmatrix}$$

Fig. 1 – Four unitary Hadamard matrices $\mathbf{H_C}$, $\mathbf{H_A}$, $\mathbf{H_T}$, and $\mathbf{H_G}$ obtained from the doubly stochastic Hermitian matrix $\mathbf{W_D}$ = [0.5, 0.5; 0.5, 0.5].

Identifying this connection between the statistical universals of genomic DNAs and these four unitary Hadamard matrices is important due to the significance of unitary transformations (unitary operators) for quantum mechanics, quantum computing, biosystems, signal processing engineering, and other fields. Unitary transformations preserve vector lengths and scalar products (preserve the metric), representing rotation and mirror reflection operators. Unitary matrices satisfy the criterion:

the product of a unitary matrix with its transpose is equal to one. Unitary transformations with real components are called orthogonal transformations, but in this article, we will use their more general name, "unitary transformations," by which they are better known in various fields of science. In quantum mechanics, unitary transformations describe the time evolution of isolated quantum systems, and in quantum mechanics (unlike classical mechanics), observable quantities are represented not by numbers, but by operators. In quantum computers, all calculations are performed on the basis of unitary operators, which act as logical gates, and any unitary operator can be used in quantum computing as a gate [15].

We will call the four unitary operators $\mathbf{H_C}$, $\mathbf{H_A}$, $\mathbf{H_T}$, and $\mathbf{H_G}$ (Fig. 1) the genetic Hadamard gates. These genetic gates are four versions of the Hadamard (2•2)-matrix with the weighting factor $2^{-0.5}$, which is traditionally used in quantum computing to transform the Hadamard matrix into a unitary operator. By definition, the Hadamard matrix H_n is a (n•n)-matrix, composed of the numbers 1 and -1, whose columns are orthogonal and the relation $H_n \bullet H^T = n \bullet E_n$ holds, where E_n is the identity matrix of order n. Among the properties of Hadamard matrices is the fact that permuting any rows and columns of the Hadamard matrix always yields a new Hadamard matrix. In passing, we note that in the general case, Hadamard matrices have many other remarkable properties and applications (see, for example, [1]).

One of these four Hadamard genetic unitary operators (gates) — $\mathbf{H_T}$ — has long been used in quantum computers for fundamental operations on qubits, serving as a key element in many quantum algorithms, including the Deutsch-Jozsa algorithm and Shor's algorithm. This Hadamard gate provides quantum algorithms with a superposition principle of quantum entanglement for quantum superiority of quantum algorithms in comparison to known classical algorithms [15].

Of the four genetic Hadamard gates, two gates $\mathbf{H_A}$ and $\mathbf{H_T}$ are mirror reflection operators. Raising them to integer powers generates the corresponding cyclic groups of unitary operators with period 2. The other two unitary matrices ($\mathbf{H_C}$, $\mathbf{H_G}$) are rotation operators ($\mathbf{H_C}$ counterclockwise, $\mathbf{H_G}$ clockwise) and matrix representations of the complex number Z = $(1+\mathbf{i}) \bullet 2^{-0.5}$, where **i** is the imaginary unit of the complex number ($i^2 = -1$). Repeated raising these unitary matrices $\mathbf{H_C}$ and $\mathbf{H_G}$ to integer powers (positive and negative) generates cyclic (with period 8) groups of unitary operators, which are matrix representations of the complex numbers (1).

$$\mathbf{H_C}^n = \mathbf{H_C}^{n+8}, \quad \mathbf{H_G}^n = \mathbf{H_G}^{n+8}, \quad \mathbf{H_A}^n = \mathbf{H_A}^{n+2}, \quad \mathbf{H_T}^n = \mathbf{H_T}^{n+2} \qquad (1)$$

Moreover, any of the unitary matrices $\mathbf{H_C}$ and $\mathbf{H_G}$ can be represented as the product of k unitary matrices, which are their kth roots. In other words, the action of a single unitary operator, for example, $\mathbf{H_C}$, can be represented as the action of a sequence of k more fractional unitary operators

$\mathbf{H_C}^{1/k}$. Thus, with the matrix-vector approach, any large transformation in the system from the action of such a large operator $\mathbf{H_C}$ can be represented as consisting of a sequence of arbitrarily small transformations from the action of the corresponding sequence of unitary operators $\mathbf{H_C}^{1/k}$ for modeling quasi-continuous transformations in the simulated cyclic processes. We also note that raising the genetic gate $\mathbf{H_C}$ or $\mathbf{H_G}$ to a power representing a cyclic function of time in its step-by-step states allows modeling quasi-continuous cyclic bioprocesses as vector sequences of their step-by-step functional states.

In quantum logic, projectors (projection operators) play an important role in quantum logic. In light of this, we note that the unitary Hadamard matrices in the H_C and H_G operators are sums of two sparse matrices representing projectors Ps satisfying the projector criterion $\mathbf{P_s}^2 = \mathbf{P_s}$ and shown in parentheses in expression (2):

$$\mathbf{H_C} = 2^{-0.5} \bullet [1\ -1;\ 1\ 1] = 2^{-0.5} \bullet ([1,\ 0;\ 1,\ 0] + [0,\ -1;\ 0,\ 1]),$$

$$\mathbf{H_G} = 2^{-0.5} \bullet [1\ 1;\ -1\ 1] = 2^{-0.5} \bullet ([1,\ 0;\ -1,\ 0] + [0,\ 1;\ 0,\ 1]) \qquad (2)$$

Many genetically inherited biological structures in organisms are clearly linked to unitary transformations of rotations and mirror images. For example, the kinematic schema of the human body and its locomotion is based on unitary transformations of rotations at the joints (the human body has approximately 300 joints) and mirror symmetry of the left and right halves of the body. Human motor activity is reduced to the skillful control by the nervous system of ensembles of these unitary transformations in body kinematics, which is associated with the genetically inherited ability of the nervous system to operate unitary transformations. Moreover, a person's very concept of their body schema is innate: people with limbs missing from birth and no personal experience using them nevertheless perceive them as truly existing, with phantom pain in them [25, 26].

When studying human sensorimotor characteristics, it's important to consider that the genetically inherited nervous system is structurally related to genetic structures. Humans see the world through probabilities in statistical streams of signals from neurons in the retina (containing millions of receptor cells) and other sensory organs. Norbert Wiener, the father of cybernetics, asserted: "*Genetic memory—the memory of our genes—is essentially determined by nucleic acid complexes...there is reason to believe that the memory of the nervous system has a similar nature*" [22, 27].

Another example of the biological importance of unitary transformations is the construction of complex three-dimensional protein shapes in the body, known as protein folding. These shapes are based on unitary transformations involving the rotation of protein molecule segments relative to each other around relatively strong carbon-carbon bonds.

The author proposes to consider and use the family of 4 genetic unitary Hadamard operators $\mathbf{H_C}$, $\mathbf{H_A}$, $\mathbf{H_T}$, $\mathbf{H_G}$ (Fig. 1) as a basic genetic quantum-logical alphabet for the development on its basis of the theory of a quantum-logical information system that allows modeling genetically inherited and logically organized biological structures and phenomena.

Here, the distinctive features of quantum logic should be clarified [3, 24]. Quantum logic is an algebraic system for describing, using quantum gates, how qubits operate and interact and how to extract information from them. In quantum logic, "logic" is not contained in reasoning, but in the mathematical description of states and operations. Quantum logic can be formulated as a modified version of propositional logic. For comparison, we recall that classical Boolean logic is a set of logical rules (AND, OR, NOT, etc.) describing how bits (0 or 1) can be combined and transformed according to the laws of Boolean algebra with its key principle of distributivity and the statements "true" or "false." Quantum logic considers not "true/false" statements, but questions to a quantum system. The answer to such a question is provided by the probability value obtained during measurement. Logical operations are replaced by quantum gates (unitary operations): NOT becomes an X gate, and completely new operations appear that have no analogues in classical logic, for example, the Hadamard gate, which creates superposition. Quantum logic operates on qubits, vectors, and matrices, not on sets. Its mathematical foundation is the theory of Hilbert spaces and projective and unitary operators. The state space of a quantum system is described by vectors, and rotations of these vectors serve as logical operations. Quantum logic lacks distributivity, which is considered its key difference from Boolean logic. Quantum logic is a branch of logic necessary for reasoning about propositions that take into account the principles of quantum theory. It was founded by the work of G. Birchow and J. von Neumann [2], who sought to reconcile the inconsistencies of classical logic with the facts about measurements in quantum mechanics and saw in quantum logic a possible foundation for physics.

The unitary Hadamard matrices of the basic operator genetic alphabet $\mathbf{H_C}$, $\mathbf{H_A}$, $\mathbf{H_T}$, $\mathbf{H_G}$ (Fig. 1) and many types of their combinations into unitary matrices of higher orders form - when they are repeatedly raised to powers - cyclic groups of operators with different periods and used to model cyclic sequences of states of quantum-like systems. The resulting algebraic-geometric apparatus is intended, first of all, for quantum-logical modeling of a set of genetically inherited cyclic and hypercyclic biostructures in genetic biomechanics. It should be noted that the quantum-logical approach to inherited cyclic and hypercyclic biostructures developed by the author, based on the alphabet of Hadamard genetic gates and the mathematical apparatus of quantum-logical biology, differs fundamentally from the well-known biochemical concept of catalytic cycles and hypercycles [4].

Global research in genetic informatics relies heavily on the fundamental fact that the DNA of all organisms contains a molecular alphabet of 4 nucleotides: C, G, A, T, and its extensions into nucleotide alphabets of 16 doublets, 64 triplets, and so on. In parallel with this molecular alphabet of 4 nucleotides, bioinformatics and genetic biomechanics now allow and should work with an operator alphabet of 4 genetic Hadamard gates, that is, a fundamentally new type of alphabet: a quantum-logical operator alphabet of 4 unitary matrices - $\mathbf{H_C}$, $\mathbf{H_A}$, $\mathbf{H_T}$, $\mathbf{H_G}$ -, which gives rise to interconnected sets of higher-order unitary operators. These genetic unitary Hadamard matrices are conjugated with corresponding complete orthogonal systems of Walsh functions. The latter are the basis of a special spectral analysis of signals in digital informatics and are associated with cyclic Gray codes, logical holography, Walsh antennas, and the fractal Hilbert curve, which is known to correspond to the spatial packing of chromatin in the human genome [13]. The relationship of Hadamard matrices with the listed areas is described in our works [16, 17, 20].

The mathematical properties of the alphabet of 4 genetic gates and the sets of unitary operators constructed on their basis are subject to systematic study. For example, the question of the existence in this set of non-commuting unitary operators whose commutators are non-zero is subject to study (in quantum mechanics, the study of pairs of operators characterized by non-zero commutators led to the formulation of the Heisenberg uncertainty principle). Already in the alphabet of 4 genetic Hadamard gates, there are three pairs of non-commuting unitary operators characterized by non-zero commutators (3); the values of these commutators are equal in two cases to Hadamard matrices, and in the third case to the matrix representation of twice the imaginary unit i of a complex number:

$$\mathbf{H_A}\bullet\mathbf{H_C} - \mathbf{H_C}\bullet\mathbf{H_A} = [1, 1; 1, -1],$$
$$\mathbf{H_C}\bullet\mathbf{H_T} - \mathbf{H_T}\bullet\mathbf{H_C} = [-1, 1; 1, 1],$$
$$\mathbf{H_A}\bullet\mathbf{H_T} - \mathbf{H_T}\bullet\mathbf{H_A} = [0, -2; 2, 0] \qquad (3)$$

At this stage of research, the question of the genetic significance and possible interpretation of these and other facts of non-zero commutators among genetic unitary operators remains open for discussion (it is possible that when analyzed from the standpoint of quantum-logical bioinformatics, it will turn out to be associated with the phenomenon of chirality in biological structures, taking into account the known facts about the existence of chirality in the quantum physics of elementary particles).

The next section discusses the development of the mathematical apparatus of quantum-logical bioinformatics, which includes a set of genetic unitary operators and their cyclic groups for modeling genetically inherited cyclic biostructures and biorhythmic processes.

3. Expansion of the set of genetic unitary operators

The DNA alphabet of 4 nucleotides C, A, T, G is identical in number of elements to the quantum logic alphabet of 4 genetic Hadamard gates $\mathbf{H_C}$, $\mathbf{H_A}$, $\mathbf{H_T}$, and $\mathbf{H_G}$ proposed by the author (Fig. 1). In matrix genetics, it is known that, based on the genetic (2•2)-matrix of the 4-nucleotide alphabet, by raising it to tensor powers, genetic $(2^n \bullet 2^n)$-matrices are formed with a strictly regular arrangement of 16 duplets, 64 triplets, 256 tetraplets, etc. [16, 20]. Figure 2 shows examples of tensor-linked genetic matrices of 4 nucleotides, 16 duplets, and 64 triplets from these books.

	0	1
0	C	A
1	T	G

;

	00	01	10	11
00	CC	CA	AC	AA
01	CT	CG	AT	AG
10	TC	TA	GC	GA
11	TT	TG	GT	GG

	000	001	010	011	100	101	110	111
000	CCC	CCA	CAC	CAA	ACC	ACA	AAC	AAA
001	CCT	CCG	CAT	CAG	ACT	ACG	AAT	AAG
010	CTC	CTA	CGC	CGA	ATC	ATA	AGC	AGA
011	CTT	CTG	CGT	CGG	ATT	ATG	AGT	AGG
100	TCC	TCA	TAC	TAA	GCC	GCA	GAC	GAA
101	TCT	TCG	TAT	TAG	GCT	GCG	GAT	GAG
110	TTC	TTA	TGC	TGA	GTC	GTA	GGC	GGA
111	TTT	TTG	TGT	TGG	GTT	GTG	GGT	GGG

Fig. 2. Tensor family of genetic matrices of 4 nucleotides C, A, T, G, 16 nucleotide duplets, and 64 nucleotide triplets.

The tensor product is a critically important operation in the quantum mechanics of multicomponent systems and quantum information science [15]. By analogy with the tensor family of genetic matrices based on the alphabet of 4 nucleotides (Fig. 2), we construct a tensor family of genetic matrices based on the alphabet of 4 Hadamard genetic gates $\mathbf{H_C}$, $\mathbf{H_A}$, $\mathbf{H_T}$, and $\mathbf{H_G}$ (Fig. 1), for example, by simply replacing the nucleotide symbols C, A, T, and G with similarly indexed symbols of these gates and connecting adjacent gates in a bundle with the tensor multiplication sign ⊗ (Fig. 3).

H_C	H_A
H_T	H_G

;

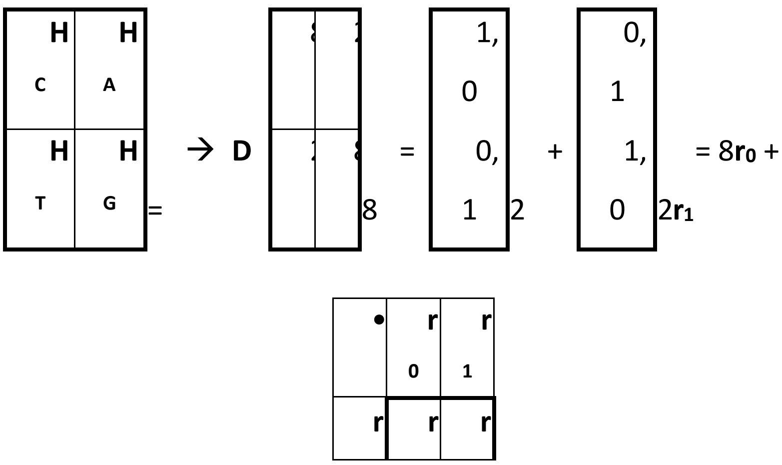
$H_C \otimes H_C$	$H_C \otimes H_A$	$H_A \otimes H_C$	$H_A \otimes H_A$
$H_C \otimes H_T$	$H_C \otimes H_G$	$H_A \otimes H_T$	$H_A \otimes H_G$
$H_T \otimes H_C$	$H_T \otimes H_A$	$H_G \otimes H_C$	$H_G \otimes H_A$
$H_T \otimes H_T$	$H_T \otimes H_G$	$H_G \otimes H_T$	$H_G \otimes H_G$

$H_C \otimes H_C \otimes H_C$	$H_C \otimes H_C \otimes H_A$	$H_C \otimes H_A \otimes H_C$	$H_C \otimes H_A \otimes H_A$	$H_A \otimes H_C \otimes H_C$	$H_A \otimes H_C \otimes H_A$	$H_A \otimes H_A \otimes H_C$	$H_A \otimes H_A \otimes H_A$
$H_C \otimes H_C \otimes H_T$	$H_C \otimes H_C \otimes H_G$	$H_C \otimes H_A \otimes H_T$	$H_C \otimes H_A \otimes H_G$	$H_A \otimes H_C \otimes H_T$	$H_A \otimes H_C \otimes H_G$	$H_A \otimes H_A \otimes H_T$	$H_A \otimes H_A \otimes H_G$
$H_C \otimes H_T \otimes H_C$	$H_C \otimes H_T \otimes H_A$	$H_C \otimes H_G \otimes H_C$	$H_C \otimes H_G \otimes H_A$	$H_A \otimes H_T \otimes H_C$	$H_A \otimes H_T \otimes H_A$	$H_A \otimes H_G \otimes H_C$	$H_A \otimes H_G \otimes H_A$
$H_C \otimes H_T \otimes H_T$	$H_C \otimes H_T \otimes H_G$	$H_C \otimes H_G \otimes H_T$	$H_C \otimes H_G \otimes H_G$	$H_A \otimes H_T \otimes H_T$	$H_A \otimes H_T \otimes H_G$	$H_A \otimes H_G \otimes H_T$	$H_A \otimes H_G \otimes H_G$
$H_T \otimes H_C \otimes H_C$	$H_T \otimes H_C \otimes H_A$	$H_T \otimes H_A \otimes H_C$	$H_T \otimes H_A \otimes H_A$	$H_G \otimes H_C \otimes H_C$	$H_G \otimes H_C \otimes H_A$	$H_G \otimes H_A \otimes H_C$	$H_G \otimes H_A \otimes H_A$
$H_T \otimes H_C \otimes H_T$	$H_T \otimes H_C \otimes H_G$	$H_T \otimes H_A \otimes H_T$	$H_T \otimes H_A \otimes H_G$	$H_G \otimes H_C \otimes H_T$	$H_G \otimes H_C \otimes H_G$	$H_G \otimes H_A \otimes H_T$	$H_G \otimes H_A \otimes H_G$
$H_T \otimes H_T \otimes H_C$	$H_T \otimes H_T \otimes H_A$	$H_T \otimes H_G \otimes H_C$	$H_T \otimes H_G \otimes H_A$	$H_G \otimes H_T \otimes H_C$	$H_G \otimes H_T \otimes H_A$	$H_G \otimes H_G \otimes H_C$	$H_G \otimes H_G \otimes H_A$
$H_T \otimes H_T \otimes H_T$	$H_T \otimes H_T \otimes H_G$	$H_T \otimes H_G \otimes H_T$	$H_T \otimes H_G \otimes H_G$	$H_G \otimes H_T \otimes H_T$	$H_G \otimes H_T \otimes H_G$	$H_G \otimes H_G \otimes H_T$	$H_G \otimes H_G \otimes H_G$

Fig. 3 – Tensor family of matrices of 4 genetic gates $\mathbf{H_C}$, $\mathbf{H_A}$, $\mathbf{H_T}$, $\mathbf{H_G}$, 16 duplets and 64 triplets of these gates.

We emphasize that the tensor product of unitary Hadamard operators always yields a unitary Hadamard operator of increased order, conjugate, as noted above, to the corresponding complete orthogonal system of Walsh functions, cyclic Gray codes, the fractal Hilbert curve, logical holography, Walsh antennas, etc. [15-17]. Accordingly, the content of each cell in the $(2^n \bullet 2^n)$-matrices of the tensor family $[\mathbf{H_C}, \mathbf{H_A}; \mathbf{H_T}, \mathbf{H_G}]^{(n)}$ represents a unitary Hadamard operator of the corresponding order. Raising each of these Hadamard gates to an integer power generates a cyclic group of unitary operators characterized by a certain period. It can be shown that the periods of all cyclic groups of gates within each of the matrices of this tensor family are interconnected based on multidimensional hypercomplex numbers. We demonstrate this using the example of the first two matrices of the tensor family under consideration, located at the top of Fig. 3.

The cyclic groups based on raising the alphabetic gates $\mathbf{H_C}$, $\mathbf{H_A}$, $\mathbf{H_T}$, $\mathbf{H_G}$ to integer powers have periods of 8 and 2, as indicated above in expression (1). Figure 4 shows a representation of the matrix $[\mathbf{H_C}, \mathbf{H_A}; \mathbf{H_T}, \mathbf{H_G}]$ from Figure 3 as a matrix **D**, which indicates the values of the periods of the cyclic groups of each of these genetic gates. It is also shown that this matrix **D** is the sum of two sparse matrices $\mathbf{r_0}$ and $\mathbf{r_1}$ with weight coefficients of 8 and 2. But the set of these matrices $\mathbf{r_0}$ and $\mathbf{r_1}$ is closed with respect to multiplication and determines a table of their multiplication (Fig. 4 below), which is known in mathematics as the multiplication table of basic elements of the algebra of 2-dimensional hyperbolic numbers [8, 16]. This means that the period matrix D is a matrix representation of the 2-dimensional hyperbolic number 8+2**j**, where **j** is the imaginary unit of the hyperbolic number.



0	0	1
r_1	r_1	r_0

Fig. 4 – Representation of the matrix of four gates $\mathbf{H_C}$, $\mathbf{H_A}$, $\mathbf{H_T}$, $\mathbf{H_G}$ from Fig. 3 in the form of a matrix **D** of periods 8 and 2 corresponding cyclic groups based on raising these gates to integer powers n: $\mathbf{H_C}^n$, $\mathbf{H_A}^n$, $\mathbf{H_T}^n$, $\mathbf{H_G}^n$.-Also shown are the decomposition of this matrix **D** into the sum of two sparse matrices $8\mathbf{r_0}$ and $2\mathbf{r_1}$ and the multiplication table of the matrices $\mathbf{r_0}$ and $\mathbf{r_1}$, which coincides with the multiplication table of the algebra of 2-dimensional hyperbolic numbers.

Let us now turn to the matrix of 16 gate tensor doublets from Fig. 3. The cyclic groups based on raising the terms of this matrix to integer powers have periods of 2, 4, and 8. Fig. 5 shows the (4•4)-matrix $\mathbf{D_2}$, representing this matrix of unitary operators as a matrix of periods of the corresponding cyclic groups of its 16 terms. This matrix $\mathbf{D_2}$ is a bisymmetric Hermitian real matrix. The sums of the components in each of its rows and columns are the same (with appropriate normalization, it becomes a doubly stochastic matrix). As shown in Fig. 5, the matrix $\mathbf{D_2}$ is the sum of 4 sparse matrices $\mathbf{s_0}$, $\mathbf{s_1}$, $\mathbf{s_2}$, $\mathbf{s_3}$ (the numbering corresponds to the order of their sequence in Fig. 5 from left to right) with weight coefficients 4, 8, 8, 2. The set of these 4 sparse matrices is closed under multiplication and determines their multiplication table, known in mathematics as the multiplication table of basis elements of the algebra of 4-dimensional hyperbolic numbers [8, 16]. This means that the period matrix of the considered cyclic groups of unitary operators $\mathbf{D_2}$ is a matrix representation of the 4-dimensional hyperbolic number $4\mathbf{s_0}+8\mathbf{s_1}+8\mathbf{s_2}+2\mathbf{s_3}$, where $\mathbf{s_0}$ is the identity matrix, and $\mathbf{s_1}$, $\mathbf{s_2}$, $\mathbf{s_3}$ are matrix representations of imaginary units of 4-dimensional hyperbolic numbers.

$$\mathbf{D_2} = \begin{bmatrix} 4 & 8 & 8 & 2 \\ 8 & 4 & 2 & 8 \\ 8 & 2 & 4 & 8 \\ 2 & 8 & 8 & 4 \end{bmatrix} = 4\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} + 8\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} + 8\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} + 2\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

•	s_0	s_1	s_2	s_3
s_0	s_0	s_1	s_2	s_3
s_1	s_1	s_0	s_3	s_2
s_2	s_2	s_3	s_0	s_1
s_3	s_3	s_2	s_1	s_0

Fig. 5 – The matrix of periods of 16 cyclic groups, representing the matrix of 16 tensor duplets in Fig. 3 and being the sum of the four shown sparse matrices $\mathbf{s_0}$, $\mathbf{s_1}$, $\mathbf{s_2}$, $\mathbf{s_3}$ with their weight coefficients. The multiplication table of these sparse matrices is shown below.

In general, such a tensor family of $(2^n \bullet 2^n)$ matrices $[\mathbf{H_C}, \mathbf{H_A}; \mathbf{H_T}, \mathbf{H_G}]^{(n)}$, obtained by raising the original alphabetic matrix of 4 genetic Hadamard gates to the tensor power (n), contains 4^n unitary operators, each of which, when raised to an integer power, generates its own cyclic group of unitary operators with a certain period. A detailed analysis of the set of these tensor-generated Hadamard unitary operators remains to be conducted in the future, including the study of their transformations under cyclic permutations of their columns and rows, as well as the study of commutators between individual operators, etc. A living organism represents a huge genetically inherited ensemble of coordinated cyclic processes occurring at all levels of biological organization: molecular, subcellular, cellular, supracellular, and organismal [17, 18]. Therefore, cyclic groups of unitary operators associated with the structural features of genetic informatics are needed for quantum-logical modeling of these ensembles of cyclic bioprocesses.

In addition to the described tensor generation of new unitary operators based on the genetic alphabet of 4 Hadamard gates, there is also another approach to their formation. It consists of constructing multi-block matrices, the blocks of which are the alphabetic Hadamard gates $\mathbf{H_C}$, $\mathbf{H_A}$, $\mathbf{H_T}$, $\mathbf{H_G}$. In this way, unitary matrices are formed, which are, in particular, matrix representations of Hamiltonian quaternions and biquaternions, which are closely related to physics, robotics, artificial intelligence, etc. Thus, thousands of papers in the 20th century alone have been devoted to quaternions and biquaternions in physics [6]. This attention to them is due to the fact that quaternions are closely related to Pauli matrices, the theory of the electromagnetic field, the quantum-mechanical theory of chemical valence, the theory of spins, the rotation of bodies in three-dimensional space, etc. Fig. 6 shows one example of such a construction, in which a unitary Hadamard matrix **Q** arises, which is a matrix representation of the Hamiltonian quaternion.

$$\mathbf{Q} = 2^{-0.5} \begin{bmatrix} \mathbf{H_G} & \mathbf{H_A} \\ -\mathbf{H_A} & \mathbf{H_G} \end{bmatrix} = 0.5 \begin{bmatrix} 1 & 1 & -1 & 1 \\ -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ -1 & -1 & -1 & 1 \end{bmatrix}$$

Fig. 6 - Unitary matrix representation **Q** of the Hamiltonian quaternion.

Indeed, as shown in Fig. 7, this matrix **Q** is the sum of four sparse matrices $\mathbf{v_0}$, $\mathbf{v_1}$, $\mathbf{v_2}$, $\mathbf{v_3}$: $\mathbf{Q} = 0.5(\mathbf{v_0} + \mathbf{v_1} + \mathbf{v_2} + \mathbf{v_3})$, where $\mathbf{v_0}$ is the identity matrix. The set of these sparse matrices is closed under multiplication and defines a multiplication table for them, which coincides with the well-known multiplication table of the basis elements of the Hamiltonian quaternion algebra [8]. This means that the matrix **Q** is a unitary matrix representation of a quaternion, where $\mathbf{v_1}$, $\mathbf{v_2}$, $\mathbf{v_3}$ represent the imaginary units of the quaternion. Quaternions that have unitary matrix representations will be called "unitary quaternions" for brevity.

$$0.5\begin{bmatrix}1, 1,-1,1\\-1, 1, 1,1\\1,-1, 1,1\\-1,-1,-1,1\end{bmatrix} = 0.5\begin{bmatrix}1,0,0,0\\0,1,0,0\\0,0,1,0\\0,0,0,1\end{bmatrix} + 0.5\begin{bmatrix}0,1, 0,0\\-1,0, 0,0\\0,0, 0,1\\0,0,-1,0\end{bmatrix} + 0.5\begin{bmatrix}0, 0,-1,0\\0, 0, 0,1\\1, 0, 0,0\\0,-1, 0,0\end{bmatrix} + 0.5\begin{bmatrix}0, 0,0,1\\0, 0,1,0\\0,-1,0,0\\-1, 0,0,0\end{bmatrix}$$

$$= 0.5(\mathbf{v_0} + \mathbf{v_1} + \mathbf{v_2} + \mathbf{v_3}).$$

•	$\mathbf{v_0}$	$\mathbf{v_1}$	$\mathbf{v_2}$	$\mathbf{v_3}$
$\mathbf{v_0}$	$\mathbf{v_0}$	$\mathbf{v_1}$	$\mathbf{v_2}$	$\mathbf{v_3}$
$\mathbf{v_1}$	$\mathbf{v_1}$	$-\mathbf{v_0}$	$\mathbf{v_3}$	$-\mathbf{v_2}$
$\mathbf{v_2}$	$\mathbf{v_2}$	$-\mathbf{v_3}$	$-\mathbf{v_0}$	$\mathbf{v_1}$
$\mathbf{v_3}$	$\mathbf{v_3}$	$\mathbf{v_2}$	$-\mathbf{v_1}$	$-\mathbf{v_0}$

Fig. 7 – The unitary Hadamard matrix $\mathbf{Q} = 0.5(\mathbf{v_0} + \mathbf{v_1} + \mathbf{v_2} + \mathbf{v_3})$ as a sum of four sparse matrices. The multiplication table of this set of sparse matrices, closed under multiplication and corresponding to the multiplication table of the Hamiltonian quaternion algebra, is shown.

Raising this unitary quaternion to integer powers $\mathbf{Q^n}$ generates a cyclic group of unitary operators with a period of 6. This cyclic group models a number of genetically inherited biological structures. For example, the regular features of human color perception, represented in Newton's 6-sector color circle for the three primary and three complementary colors (Fig. 8), correspond to the cyclic group of the unitary quaternion $\mathbf{Q^n}$; its period contains 6 terms, the algebraic relationships of which correspond to the relationships of these colors in human color perception:

- 1) colors opposite on the circle cancel each other out when superimposed (just like unitary quaternion matrices opposite on the circle, whose addition yields the zero matrix);

- 2) each color on the circle is the sum of the two colors on its sides (the same is true for the corresponding unitary quaternions);

- 3) the three primary colors, like the three complementary colors at the vertices of the two triangles of the "Star of David," cancel each other out when superimposed (similarly, the sum of the unitary quaternions at the vertices of each of the two triangles of the "Star of David" is zero).

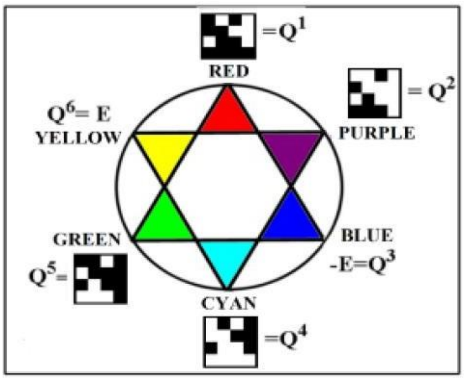


Fig. 8 - Newton's color circle from the psychophysics of color perception and the correspondence to it of the members of the matrix cyclic group of the unitary Hamilton quaternion $\mathbf{Q^n}$, the period of which is equal to 6. In each shown matrix, the black cells contain the numbers "+0.5", and the white ones - "-0.5" (the image is taken from the author's book [20]).

In psychophysics, it is well known that color is not a physical property of an object, but an inherited psychophysical response of a person to the light stimuli coming from the object. In modern literature, the characteristics of color perception, like other characteristics of sensory experience, are referred to as "qualia." The topic of qualia is one of the most pressing and widely discussed in modern philosophy, which sees it as the key to understanding the nature of consciousness. In light of this, it seems significant that the described quantum-logical bioinformation system, based on the genetic alphabet of Hadamard unitary operators, enables algebraic modeling of the properties of human color perception. The author believes that mastering quantum-logical knowledge of "qualia structures"—the inherited structural regularities of sensory experience—allows one to conceptualize and develop qualia technologies for medical, biotechnological, and agricultural purposes.

The limited space of this article does not allow for the presentation of numerous other results and examples of quantum-logical modeling of genetically inherited biostructures based on the alphabet of genetic gates $\mathbf{H_C}$, $\mathbf{H_A}$, $\mathbf{H_T}$, $\mathbf{H_G}$. These will be covered by the author later in a larger publication.

Some concluding remarks

In a quantum-logical approach to bioinformatics, taking into account the cyclical (or pulsating) properties of living bodies, the author relies on a model of biomechanical environments consisting of interconnected pulsating structures that change in a coordinated manner over time. The theory of such model software environments can be used in the development of artificial intelligence, including in connection with systolic processors and pulsating information lattice (pulser) architectures known in computer technology [7, 10]. The name "pulsating" reflects the essence of this architecture, traditionally compared to a heartbeat or pulse. The pulsation appears as a wave of data, and the computational process appears as the propagation of waves of activity. Data received at the lattice inputs begins to "pulsate" through it, being transformed at each step. The lattice can be configured so that different data streams collide and interact in specific cells at strictly defined intervals, generating a new "pulse" of results.

This architecture is fundamentally different from the von Neumann architecture of conventional processors because it has no central control unit; all cells operate simultaneously and synchronously; data is not written in the classical sense, but continuously "pulses" through the processor structure, like the flow of blood through capillaries. The operation of such a pulsating information grid is compared to the work of the heart: the grid is compared to the muscle tissue of the myocardium; the processing elements are compared to individual muscle cells of the heart (cardiomyocytes); the heartbeat is compared to the electrical impulse from the sinoatrial node; computation is compared to the coordinated contraction of the heart pumping blood; information data is compared to the pumped blood. Moreover, "computation" (pumping blood) is an emergent property of the entire organ, pulsating in a coordinated rhythm; no individual cell is responsible for it, but all cells follow the general rhythm and local interactions. Significant challenges in programming and hardware have prevented pulsating information grids from becoming widespread. The most famous example of the pulsir concept is the Connection Machine, developed by Thinking Machines Corporation in the 1980s. The pulsir concept is also closely related to a number of modern architectural concepts, such as the concept of systolic arrays, named for their pulse-like behavior similar to cardiac systole [11, 12]. Bio-inspired systolic arrays are extremely effective in artificial intelligence, image processing, pattern recognition, computer vision, and other tasks. An example is Google's Tensor Processing Unit (TPU), which uses a large two-dimensional systolic array to perform the extensive matrix multiplications required by neural networks with high efficiency.

Quantum logic in the theory of Hilbert spaces deals not only with unitary operators but also with projection operators, which serve as measurement tools and which are also represented in the inherited functions of living things. An example is the structure of vision based on the projection of light rays onto the retina. One should note that a set of projection matrices **P**, which satisfy the projectors criterion $\mathbf{P} = \mathbf{P}^2$, is associated with the genetic alphabet of unitary Hadamard matrices for example: $\mathbf{P} = 2^{-0.5}[\mathbf{H_C}, -\mathbf{H_A}; -\mathbf{H_A}, \mathbf{H_C}] = 2^{-1}[1,-1,1,-1; 1, 1,-1,-1; 1,-1,1,-1; -1,-1,1,1]$.

The quantum-logical bioinformatics system presented in this article, based on alphabets of unitary operators, is useful for explaining why complex organisms evolved so rapidly: complex organs and tissues are formed not so much by the emergence of new genes, but by changes in the ways existing genes are used under the influence of quantum-logical operators. The author calls it as "operator quantum-logical Darwinism," according to which natural selection and the inheritance of the most useful ensembles of quantum-logical operators play an important role in biological evolution.

This quantum-logical bioinformatics appears to be useful for the analysis of gene networks and related problems [9]. An important component of the formalisms of this quantum-logical informatics are tensor-unitary transformations, which provide an increase in the dimensionality of configurational vector Hilbert spaces in quantum-logical models of growing biosystems [19].

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