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Formalisms for conceptual design of closed information systems*

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A closed information system is an information system such that its environment does not change it, and there is an information transfer from it to its environment and from its environment to it. In this paper two formalisms (information query systems and conceptual configuration systems) for abstract unified modelling of the artifacts (concept sketches and models) of the conceptual design of closed information systems, early phase of information systems design process, are proposed. Information query systems defines the abstract unified information model for the artifacts, based on such general concepts as state, information query and answer. Conceptual configuration systems are a formalism for conceptual modelling of information query systems. They defines the abstract unified conceptual model for the artifacts. The basic definitions of the theory of conceptual configuration systems are given. These systems were demonstrated to allow to model both typical and new kinds of ontological elements. The classification of ontological elements based on such systems is described. A language of conceptual configuration systems is defined.

Keywords: closed information system, information query system, conceptual structure, ontology, ontological element, conceptual, conceptual state, conceptual configuration, conceptual configuration system, conceptual information query model, CCSL

1. Introduction

The conceptual models play an important role in the overall system development life cycle [1]. Numerous conceptual modelling techniques have been created, but all of them have a limited number of kinds of ontological elements and therefore can only represent ontological elements of fixed conceptual granularity. For example, entity-relationship modelling technique [2] uses two kinds of ontological elements: entities and relationships.

The purpose of the paper is propose formalisms for abstract unified modelling of the artifacts (concept sketches and models) of the conceptual design of closed information systems (IS for short) by ontological elements of arbitrary conceptual granularity. In our two stage approach the informational and conceptual aspects of the system that the conceptual model represents are

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described by two separate formalisms. The first formalism describes the informational model of the system, and the second formalism describes the conceptual model of the informational model.

The first formalism called an information query system (IQS for short) is a system characterized by sets of states, state objects, information queries, information query objects, answers, answer objects and an interpretation function. States of an IQS models the information storage in an IS modelled by the IQS, queries of the IQS model the information transferring from an environment to the IS to get the storage content, and answers of the IQS model the information transferring from the IS to the environment initiated by these queries. State objects, query objects and answer objects are objects that can be observed in states, queries and answers, respectively. They describe the observed internal structure of states, queries and answers. The interpretation function models the information transfer from the IS to its environment and from its environment to the IS. It associates queries with functions from states to answers.

A wide variety of information systems is modelled by IQSs in the information aspect, including search services with search results as answers, factual factographic databases with factual information as answers, document databases with documents as answers, content consumption devices with content information as answers, logical systems with truth values as answers, formalisms specifying denotational semantics of programming languages with denotations as answers and so on.

We consider that the second formalism used for for conceptual modelling of IQSs must meet the following general requirements (in relation to modelling of a IQS):

1. It must model the conceptual structure of states and state objects of the IQS.
2. It must model the content of the conceptual structure.
3. It must model information queries, information query objects, answers and answer objects of the IQS.
4. It must model the interpretation function of the IQS.
5. It must be quite universal to model typical ontological elements (concepts, attributes, concept instances, relations, relation instances, types, domains, and so on.).
6. It must provide a quite complete classification of ontological elements, including the determination of their new kinds and subkinds with arbitrary conceptual granularity.
7. The model of the interpretation function must be extensible.
8. It must have language support. The language associated with the formalism must define

syntactic representations of models of states, state objects, queries, query objects, answers and answer objects and includes the set of predefined basic query models.

To our knowledge, there is no formalism that meets all the above requirements. Therefore, we propose a new formalism, conceptual configuration systems (CCS for short), that meets these requirements.

The paper has the following structure. The preliminary concepts and notation are given in section 2. The formal definition of IQSs and the basic definitions of the theory of CCSs are given in section 3. The structure of conceptuals (atomic conceptual structures of CCSs) is described in section 4. The structure of conceptual states is considered in section 5. The classification of elements of conceptual states such that concepts, attributes and individuals is presented in section 6. The structure of concepts is described in section 7. The classification and interpretation of concepts is given in 8. The structure of attributes is described in section 9. The classification and interpretation of attributes is given in 10. The classification of conceptuals and ontological elements modelled by these conceptuals is presented in section 11. Relations, types, domains and inheritance are modelled by conceptual structures of CCSs in section 12. Generic conceptuals describing sets of conceptuals satisfying a pattern are defined in section 13. The language CCSL of CCSs is described in section 14. The semantics of interpretable elements in CCSL is defined in section 15. We establish that CCSs meet the above requirements in section 16. CCSs are compared with the related formalism, abstract state machines [3, 4], in section 17.

2. Preliminaries

2.1. Sets, sequences, multisets

Let O_b be the set of objects considered in this paper. Let S_t be a set of sets. Let I_{nt}, N_t, N_{t0} and B_l be sets of integers, natural numbers, natural numbers with zero and boolean values $true$ and $false$, respectively.

Let the names of sets be represented by capital letters possibly with subscripts and the elements of sets be represented by the corresponding small letters possibly with extended subscripts. For example, i_{nt} and $i_{nt.1}$ are elements of I_{nt}.

Let S_q be a set of sequences. Let $s_{t.(*)}$, $s_{t.\{*\}}$, and $s_{t.*}$ denote sets of sequences of the forms $(o_{b.1}, \ldots, o_{b.n_{t0}})$, $\{o_{b.1}, \ldots, o_{b.n_{t0}}\}$, and $o_{b.1}, \ldots, o_{b.n_{t0}}$ from elements of s_t. For example, $I_{nt.(*)}$ is a set of sequences of the form $(i_{nt.1}, \ldots, i_{nt.n_{t0}})$, and $i_{nt.*}$ is a sequence of the form $i_{nt.1}, \ldots, i_{nt.n_{t0}}$.

Let $o_{b.1}, \ldots, o_{b.n_{t0}}$, denote $o_{b.1}, \ldots, o_{b.n_{t0}}$. Let $s_{t.(*n_{t0})}$, $s_{t.\{*n_{t0}\}}$, and $s_{t.*n_{t0}}$ denote sets of the corresponding sequences of the length n_{t0}.

Let $o_{b.1} \prec_{[\![s_q]\!]} o_{b.2}$ denote the fact that there exist $o_{b.*.1}$, $o_{b.*.2}$ and $o_{b.*.3}$ such that $s_q = o_{b.*.1},\ o_{b.1},\ o_{b.*.2},\ o_{b.2},\ o_{b.*.3}$, or $s_q = (o_{b.*.1},\ o_{b.1},\ o_{b.*.2},\ o_{b.2},\ o_{b.*.3})$.

Let $[o_b\ o_{b.1} \hookleftarrow o_{b.2}]$ denote the result of replacement of all occurrences of $o_{b.1}$ in o_b by $o_{b.2}$. Let $[s_q\ o_b \hookleftarrow_* o_{b.1}]$ denote the result of replacement of each element $o_{b.2}$ in s_q by $[o_{b.1}\ o_b \hookleftarrow o_{b.2}]$. For example, $[(a, b)\ x \hookleftarrow_* (f\ x)]$ denotes $((f\ a), (f\ b))$.

Let $[len\ s_q]$ denote the length of s_q. Let und denote the undefined value. Let $[s_q\ .\ n_t]$ denote the n_t-th element of s_q. If $[len\ s_q] < n_t$, then $[s_q\ .\ n_t] = und$. Let $[s_q\ +\ s_{q.1}]$, $[o_b\ .+\ s_q]$ and $[s_q\ +.\ o_b]$ denote $o_{b.*}, o_{b.*.1}$, $o_b, o_{b.*}$ and $o_{b.*}, o_b$, where $s_q = o_{b.*}$ and $s_{q.1} = o_{b.*.1}$.

Let $[and\ s_q]$ denote $(c_{nd.1}\ and\ \ldots\ and\ c_{nd.n_t})$, where $s_q = c_{nd.1}$, ..., $c_{nd.n_t}$, and $[and]$ denote $true$. In the case of $n_t = 1$, the brackets can be omitted.

Let $o_{b.1}, o_{b.2} \in S_t \cup S_q$. Then $o_{b.1} =_{st} o_{b.2}$ denote that the sets of elements of $o_{b.1}$ and $o_{b.2}$ coincide, and $o_{b.1} =_{ml} o_{b.2}$ denote that the multisets of elements of $o_{b.1}$ and $o_{b.2}$ coincide.

2.2. Contexts

The terms used in the paper are context-dependent.

Let L_b be a set of objects called labels. Contexts have the form $[\![o_{b.*}]\!]$, where the elements of $o_{b.*}$ called embedded contexts have the form: l_b:o_b, l_b: or o_b.

The context in which some embedded contexts are omitted is called a partial context. All omitted embedded contexts are considered bound by the existential quantifier, unless otherwise specified.

Let $o_b[\![o_{b.*}]\!]$ denote the object o_b in the context $[\![o_{b.*}]\!]$.

The object 'in $[\![o_b, o_{b.*}]\!]$' can be reduced to 'in $[\![o_b]\!]$ in $[\![o_{b.*}]\!]$' if this does not lead to ambiguity.

2.3. Functions

Let F_n be a set of functions. Let A_{rg} and V_l be sets of objects called arguments and values. Let $[f_n\ a_{rg.*}]$ denote the application of f_n to $a_{rg.*}$.

Let $[support\ f_n]$ denote the support in $[\![f_n]\!]$, i. e. $[support\ f_n] = \{a_{rg} : [f_n\ a_{rg}] \neq und\}$. Let $[image\ f_n\ s_t]$ denote the image in $[\![f_n, s_t]\!]$, i. e. $[image\ f_n\ s_t] = \{[f_n\ a_{rg}] : a_{rg} \in s_t\}$. Let $[image\ f_n]$ denote the image in $[\![f_n, [support\ f_n]]\!]$. Let $[narrow\ f_n\ s_t]$ denote the function $f_{n.1}$ such that $[support\ f_{n.1}] = [support\ f_{n.1}] \cap s_t$, and $[f_{n.1}\ a_{rg}] = [f_n\ a_{rg}]$ for each $a_{rg} \in [support\ f_{n.1}]$.

The function $f_{n.1}$ is called a narrowing of f_n to s_t. Let $[support\ f_{n.1}] \cap [support\ f_{n.2}] = \emptyset$. Let $f_{n.1} \cup f_{n.2}$ denote the union f_n of $f_{n.1}$ and $f_{n.2}$ such that $[f_n\ a_{rg}] = [f_{n.1}\ a_{rg}]$ for each $a_{rg} \in [support\ f_{n.1}]$, and $[f_n\ a_{rg}] = [f_{n.2}\ a_{rg}]$ for each $a_{rg} \in [support\ f_{n.2}]$. Let $f_{n.1} \subseteq f_{n.2}$ denote the fact that $[support\ f_{n.1}] \subseteq [support\ f_{n.2}]$, and $[f_{n.1}\ a_{rg}] = [f_{n.2}\ a_{rg}]$ for each $a_{rg} \in [support\ f_{n.1}]$.

An object u_p of the form $a_{rg} : v_l$ is called an update. Let U_p be a set of updates. The objects a_{rg} and v_l are called an argument and value in $[\![u_p]\!]$.

Let $[f_n\ u_p]$ denote the function $f_{n.1}$ such that $[f_{n.1}\ a_{rg}] = [f_n\ a_{rg}]$ if $a_{rg} \neq a_{rg}[\![u_p]\!]$, and $[f_{n.1}\ a_{rg}[\![u_p]\!]] = v_l[\![u_p]\!]$. Let $[f_n\ u_p, u_{p.*n_t}]$ be a shortcut for $[[f_n\ u_p]\ u_{p.*n_t}]$. Let $[f_n\ a_{rg}.a_{rg.1}.\ \ldots\ .a_{rg.n_t} : v_l]$ be a shortcut for $[f_n\ a_{rg} : [[f_n\ a_{rg}]\ a_{rg.1}.\ \ldots\ .a_{rg.n_t} : v_l]]$. Let $[u_{p.*}]$ be a shortcut for $[f_n\ u_{p.*}]$, where $[support\ f_n] = \emptyset$.

Let C_{nd} be a set of objects called conditions. Let $[if\ c_{nd}\ then\ o_{b.1}\ else\ o_{b.2}]$ denote the object o_b such that

- if $c_{nd} = true$, then $o_b = o_{b.1}$;
- if $c_{nd} = false$, then $o_b = o_{b.2}$.

2.4. Attributes and multi-attributes

An object $o_{b.ma}$ of the form $(u_{p.*})$ is called a multi-attribute object. Let $O_{b.ma}$ be a set of multi-attribute objects. The elements of $[o_{b.ma}\ w \hookleftarrow_*\ a_{rg}[\![w]\!]]$ are called multi-attributes in $[\![o_{b.ma}]\!]$. Let $O_{b.ma}$ be a set of multi-attributes. The elements of $[o_{b.ma}\ w \hookleftarrow_*\ v_l[\![w]\!]]$ are called values in $[\![o_{b.ma}]\!]$. The sequence $u_{p.*}$ is called a sequence in $[\![o_{b.ma}]\!]$ and denoted by $[sequence\ in\ o_{b.ma}]$. An object v_l is a value in $[\![a_{tt.m}, o_{b.ma}]\!]$ if $o_{b.ma} = (u_{p.*.1}, a_{tt.m} : v_l, u_{p.*.2})$ for some $u_{p.*.1}$ and $u_{p.*.2}$.

An object $o_{b.ma}$ is an attribute object if the elements of $[o_{b.ma}\ w \hookleftarrow_*\ a_{rg}[\![w]\!]]$ are pairwise distinct. Let $O_{b.a}$ be a set of attribute objects. The multi-attributes in $[\![o_{b.a}]\!]$ are called attributes in $[\![o_{b.a}]\!]$. Let A_{tt} be a set of objects called attributes.

Let $[function\ o_{b.a}]$, $[o_{b.a}\ a_{tt}]$, and $[support\ o_{b.a}]$ denote $[[sequence\ in\ o_{b.a}]]$, $[[function\ o_{b.a}]\ a_{tt}]$, and $[support\ [function\ o_{b.a}]]$.

Let $[seq{-}to{-}att{-}obj\ s_q]$ denote $(1 : [s_q\ .\ 1], ..., [len\ s_q] : [s_q\ .\ [len\ s_q]])$. Let $o_{b.a} =_{st} (1 : v_{l.1}, ..., n_t : v_{l.n_t})$. Then $[att{-}obj{-}to{-}seq\ o_{b.a}]$ denote $(v_{l.1}, ..., v_{l.n_t})$.

3. Basic definitions of the theory of conceptual configuration systems

3.1. Information query systems

Let S_{tt} be a state of objects called states. An object $s_{s.q.i}$ of the form $(S_{tt}, O_{b.s}, Q_r, O_{b.q}, A_{ns}, O_{b.a}, value)$ is an information query system if S_{tt}, $O_{b.s}$, Q_r, $O_{b.q}$, A_{ns} and $O_{b.a}$ are nonempty sets, $S_{tt} \subseteq O_{b.s}$, $Q_r \subseteq O_{b.q}$, $und \in A_{ns}$, $A_{ns} \subseteq O_{b.a}$, $value \in Q_r \times S_{tt} \to A_{ns}$, and for all $q_r \in Q_r$ there exists $s_{tt} \in S_{tt}$ such that $[value\ q_r\ s_{tt}] \neq und$. Let $S_{s.q.i}$ be a set of information query systems.

The elements of S_{tt}, $O_{b.s}$, Q_r, $O_{b.q}$, A_{ns} and $O_{b.a}$ are called states, state objects, information queries, information query objects, answers and answer objects in $[\![s_{s.q.i}]\!]$, respectively. The function $value$ is called a query interpretation in $[\![s_{s.q.i}]\!]$. An object $o_{b.s}$ is a proper state object if $o_{b.s} \notin S_{tt}$. An object $o_{b.q}$ is a proper query object if $o_{b.q} \notin Q_r$. An object $o_{b.a}$ is a proper answer object if $o_{b.a} \notin A_{ns}$.

As a through illustrative example of the IQS modelled by CCSs we use the geometric system that includes the following proper state objects:

- kinds of geometric spaces (Euclidean, Riemannian, Lobachevskian and so on);
- kinds of geometric figures (triangles, rectangles, cubes and so on);
- numerical characteristics of geometric figures (length, area, volume and so on);
- units of measurement of numerical characteristics (inches, centimeters, metres and so on);
- values of numerical characteristics represented by real numbers;
- numeral systems for representing values of numerical characteristics (binary, octal, decimal and so on);
- dimensions of geometric spaces represented by natural numbers;
- named geometric figures represented by elements of the set F_g).

A state of the geometric system is a set of relations between proper state objects. For example, the relation $\{figure : f_g, kind : triangle, space : Euclidean\}$ in $[\![s_{tt}]\!]$ means that f_g is a triangle in Euclidean space in $[\![s_{tt}]\!]$, the relation $\{figure : f_g, characteristic : perimeter, value : 20\}$ in $[\![s_{tt}]\!]$ means that perimeter of f_g equals 20 in $[\![s_{tt}]\!]$, and the relation $\{kind : cube, space : Euclidean, characteristic : volume, unit : inch)$ in $[\![s_{tt}]\!]$ means volume of cubes in Euclidean space measured in inches in $[\![s_{tt}]\!]$.

The possible queries in the geometric system can be "area of f_g", "f_g is a triangle" and "unit of measurement of perimeter of f_g" returning a number, boolean value and unit of measurement as answers.

3.2. Atoms

A set A_{tm} is a set of atoms if $I_{nt} \cup \{true, und\} \subseteq A_{tm}$. Structures of CCSs are constructed from atoms. Therefore, they are implicitly defined in $[\![A_{tm}]\!]$.

⨁ Let $F_g \subseteq A_{tm}$.

3.3. Elements

Elements are basic structures of CCSs. They model query objects, answer objects and some proper state objects of IQSs. Let E_l be a set of objects called elements. An object e_l of the forms a_{tm}, $e_{l.(*)}$, $e_{l.1} : e_{l.2}$, or $e_{l.1} :: e_{l.2}$ is called an element.

An element $e_{l.(*)}$ of the form $(e_{l.*})$ is called a sequence element. The object $e_{l.*}$ is called a sequence in $[\![e_{l.(*)}]\!]$ and denoted by $[sequence\ in\ e_{l.(*)}]$. The element () is called an empty element.

An element $u_{p.e}$ of the form $a_{tt} : v_l$ is called an element update. Let $U_{p.e}$ be a set of element updates. The elements a_{tt} and v_l are called an attribute and value in $[\![u_{p.e}]\!]$.

Let S_{rt} be a set of objects called sorts. An element $e_{l.s}$ of the form $e_l :: s_{rt}$ is called a sorted element. Let $E_{l.s}$ be a set of sorted elements. The elements e_l and s_{rt} are called an element and sort in $[\![e_l]\!]$.

An element e_{xc} of the form $e_l :: exc$ is called an exception. Let E_{xc} be a set of exceptions. The element e_l is called a value in $[\![e_{xc}]\!]$. Thus, the sort exc specifies exceptions. Exceptions in CCSs play the role that is analogous to the role of exceptions in programming languages. An element e_l is abnormal if $e_l \in E_{xc}$, or $e_l = und$. Let $E_{l.ab}$ be a set of abnormal elements. An element e_l is normal if e_l is not abnormal. Let $E_{l.n}$ be a set of normal elements.

An element $e_{l.ma}$ is a multi-attribute element if $e_l \in O_{b.ma}$. Let $E_{l.ma}$ be a set of multi-attribute elements. An element $e_{l.a}$ is an attribute element if $e_l \in O_{b.a}$. Let $E_{l.a}$ be a set of attribute elements.

⨁ The element $(f_g,\ is,\ triangle)$ means that f_g is a triangle.

3.4. Conceptuals

Conceptuals are atomic conceptual structures of CCSs. Conceptual structures of CCSs are constructed from conceptuals. Conceptuals model some proper state objects of IQSs. An attribute element c_{ncpl} is a conceptual if $[support\ c_{ncpl}] \subseteq I_{nt}$. Let C_{ncpl} be a set of conceptuals. An element of the form $i_{nt} : e_l$ is called a conceptual update. Let $U_{p.c}$ be a set of conceptual updates.

⨁ Let $c_{ncpl} = (-3 : 10, -2 : inch, -1 : area, 0 : f_g, 1 : triangle, 2 : Euclidean, 3 : 2)$. Then the following properties hold:

- c_{ncpl} is a conceptual;
- $-3 : 10$, $-2 : inch$, $-1 : area$, $0 : f_g$, $1 : triangle$, $2 : Euclidean$ and $3 : 2$ are conceptual updates;
- c_{ncpl} models the area (the attribute -1) of the triangle (the attribute 1) f_g (the attribute 0) in three-dimensional (the attribute 3) Euclidean (the attribute 2) space, measured in inches (the attribute -2) in the decimal system (the attribute -3).

3.5. Conceptual states

Conceptual states are conceptual structures of CCSs specifying values of conceptuals. They model some proper state objects of IQSs. An attribute element s_{tt} is a conceptual state if $[support\ s_{tt}] \subseteq C_{ncpl}$. Thus, s_{tt} can reference to either a state of a IQS or a conceptual state of a QTS depending on the context.

A function $value \in C_{ncpl} \times S_{tt} \to E_l$ is a conceptual interpretation if $[value\ c_{ncpl}\ s_{tt}] = [s_{tt}\ c_{ncpl}]$. The element $[value\ c_{ncpl}\ s_{tt}]$ is called a value in $[\![c_{ncpl}, s_{tt}]\!]$.

⨁ Let $c_{ncpl} = (-3 : 10, -2 : inch, -1 : area, 0 : f_g, 1 : triangle, 2 : Euclidean, 3 : 2)$ and $s_{tt} = (c_{ncpl} : 3)$. Then the following properties hold:

- $[value\ c_{ncpl}\ s_{tt}] = 3$;
- 3 is the value in $[\![c_{ncpl}, s_{tt}]\!]$;
- area of the triangle f_g in two-dimensional Euclidean space equals 3 inches in the decimal system in $[\![s_{tt}]\!]$.

3.6. Conceptual configurations

Conceptual configurations are conceptual structures of CCSs partitioning states into named substates. They model states of IQSs. Let N_m be a set of objects called names. An attribute element c_{nf} is a conceptual configuration if $[image\ c_{nf}] \subseteq S_{tt}$. Let C_{nf} be a set of configurations. An element n_m is a name in $[\![c_{nf}]\!]$ if $n_m \in [support\ c_{nf}]$. An element n_m is a name in $[\![s_{tt}, c_{nf}]\!]$ if $[c_{nf}\ n_m] = s_{tt}$. An element s_{tt} is a substate in $[\![c_{nf}]\!]$ if $s_{tt} \in [image\ c_{nf}]$. An element s_{tt} is a substate in $[\![n_m, c_{nf}]\!]$ if $[c_{nf}\ n_m] = s_{tt}$. A substate s_{tt} is unnamed in $[\![c_{nf}]\!]$ if $[c_{nf}\ ()] = s_{tt}$. The element () is called an unnamed substate specifier.

A function $value \in C_{ncpl} \times E_l \times C_{nf} \to E_l$ is a conceptual interpretation if $[value\ c_{ncpl}\ n_m$

$c_{nf}] = [value\ c_{ncpl}\ [c_{nf}\ n_m]]$. The element $[value\ c_{ncpl}\ n_m\ c_{nf}]$ is called a value in $[\![c_{ncpl}, n_m, c_{nf}]\!]$.

An element $s_{tt.n}$ of the form $s_{tt} :: state :: n_m$ is called a named state. Let $S_{tt.n}$ be a set of named states. The elements s_{tt} and n_m are called a state and name in $[\![s_{tt.n}]\!]$. The element s_{tt} references to $s_{tt} :: state :: ()$ in the context of named states.

An element $c_{ncpl.n}$ of the form $c_{ncpl} :: state :: n_m$ is called a named conceptual. Let $C_{ncpl.n}$ be a set of named conceptuals. It specifies the conceptual c_{ncpl} in the state with the name n_m. The elements c_{ncpl} and n_m are called a conceptual and name in $[\![c_{ncpl.n}]\!]$. The element c_{ncpl} references to $c_{ncpl} :: state :: ()$ in the context of named conceptuals.

A function $value \in C_{ncpl.n} \times C_{nf} \to E_l$ is a conceptual interpretation if $[value\ c_{ncpl.n}\ c_{nf}] = [value\ c_{ncpl}[\![c_{ncpl.n}]\!]\ n_m[\![c_{ncpl.n}]\!]\ c_{ncpl}]$. The element $[value\ c_{ncpl.n}\ c_{nf}]$ is called a value in $[\![c_{ncpl.n}, c_{nf}]\!]$.

3.7. Substitutions, patterns, pattern specifications, instances

A function $s_b \in E_l \to E_{l.*}$ is called a substitution. Let S_b be a set of substitutions. A function $subst \in S_b \times E_{l.*} \to E_{l.*}$ is a substitution function if it is defined as follows (the first proper rule is applied):

- if $e_l \in [support\ s_b]$, then $[subst\ s_b\ e_l] = [s_b\ e_l]$;
- $[subst\ s_b\ a_{tm}] = a_{tm}$;
- $[subst\ s_b\ l_b : e_l] = [subst\ s_b\ l_b] : [subst\ s_b\ e_l]$;
- $[subst\ s_b\ e_l :: nosubst] = e_l$;
- $[subst\ s_b\ e_l :: (nosubstexcept\ e_{l.*})] = [subst\ [narrow\ s_b\ \{e_{l.*}\}]\ e_l]$;
- $[subst\ s_b\ e_l :: s_{rt}] = [subst\ s_b\ e_l] :: [subst\ s_b\ s_{rt}]$;
- $[subst\ s_b\ (e_{l.*})] = ([e_{l.*}\ w \hookleftarrow_* [subst\ s_b\ w]])$;
- $[subst\ s_b\ e_{l.*}] = [e_{l.*}\ w \hookleftarrow_* [subst\ s_b\ w]]$.

The sort $nosubst$ specifies the elements to which the substitution s_b is not applied. The sort $(nosubstexcept\ e_{l.*})$ specifies the elements to which the narrowing of the substitution s_b to the set $e_{l.*}$ is applied. An element p_t is a pattern in $[\![e_l, s_b]\!]$ if $[subst\ s_b\ p_t] = e_l$. Let P_t be a set of patterns. An element i_{nst} is an instance in $[\![p_t, s_b]\!]$ if $[subst\ s_b\ p_t] = i_{nst}$. Let I_{nst} be a set of instances.

Let V_r and $V_{r.s}$ be sets of objects called element variables and sequence variables, respectively. An element $p_{t.s}$ of the form $(p_t, (v_{r.*}), (v_{r.s.*}))$ is a pattern specification if $\{v_{r.s.*}\} \cap \{v_{r.*}\} = \emptyset$, and the elements of $\{v_{r.*}\} \cup \{v_{r.s.*}\}$ are pairwise distinct. Let $P_{t.s}$ be a set of pattern specifications.

The objects p_t, $(v_{r.*})$, and $(v_{r.s.*})$ are called a pattern, element variable specification, and sequence variable specification in $[\![p_{t.s}]\!]$. The elements of $v_{r.*}$ and $v_{r.s.*}$ are called element pattern variables and sequence pattern variables in $[\![p_{t.s}]\!]$, respectively.

An element i_{nst} is an instance in $[\![p_{t.s}, s_b]\!]$ if $[support\ s_b] = \{v_{r.*}\}$, $[s_b\ v_r] \in E_l$ for $v_r \in \{v_{r.*}\} \setminus \{v_{r.s.*}\}$, $[s_b\ v_r] \in E_{l.*}$ for $v_r \in \{v_{r.s.*}\}$, and i_{nst} is an instance in $[\![p_t, s_b]\!]$. An element i_{nst} is an instance in $[\![p_{t.s}]\!]$ if there exists s_b such that i_{nst} is an instance in $[\![p_{t.s}, s_b]\!]$.

A function $m_t \in E_l \times P_{t.s} \to S_b$ is a match if the following property holds:

- if $[m_t\ e_l\ p_{t.s}] = s_b$, then e_l is an instance in $[\![p_{t.s}, s_b]\!]$.

An element i_{nst} is an instance in $[\![p_{t.s}, m_t, s_b]\!]$ if $[m_t\ i_{nst}\ p_{t.s}] = s_b$. An element i_{nst} is an instance in $[\![p_{t.s}, m_t]\!]$ if there exists s_b such that i_{nst} is an instance in $[\![p_{t.s}, m_t, s_b]\!]$.

3.8. The element interpretation

Queries and answers of a IQS is modelled by elements, and the query interpretation of the IQS is modelled by the element interpretation $value \in E_l \times C_{nf} \to E_l$ based on atomic element interpretations, element definitions and the element interpretation order.

The special variable $conf :: in$ references to the current configuration in the definitions below.

An object $i_{ntr.a}$ of the form $(p_t, (v_{r.*}), (v_{r.s.*}), f_n)$ is an atomic element interpretation if $(p_t, (v_{r.*}), (v_{r.s.*}))$ is a pattern specification, $conf :: in \notin \{v_{r.*}\} \cup \{v_{r.s.*}\}$, $f_n \in S_b \to E_l$, $[support\ f_n] = \{s_b : [support\ s_b] = \{v_{r.*}\} \cup \{v_{r.s.*}\} \cup \{conf :: in\}, [s_b\ v_r] \in E_l$ for $v_r \in \{v_{r.*}\}$, and $[s_b\ v_r] \in E_{l.*}$ for $v_r \in \{v_{r.s.*}\}\}$. Let $I_{ntr.a}$ be a set of atomic element interpretations.

The objects p_t, $(v_{r.*})$, $(v_{r.s.*})$, and f_n are called a pattern, element variable specification, sequence variable specification, and value in $[\![i_{ntr.a}]\!]$. The elements of $v_{r.*}$ and $v_{r.s.*}$ are called element pattern variables and sequence pattern variables in $[\![i_{ntr.a}]\!]$, respectively.

A function $i_{ntr.a.s} \in E_l \to I_{ntr.a}$ is called an atomic element interpretation specification if $[support\ i_{ntr.a.s}]$ is finite. An interpretation $i_{ntr.a}$ is an atomic element interpretation in $[\![i_{ntr.a.s}]\!]$ if $[i_{ntr.a.s}\ n_m] = i_{ntr.a}$ for some $n_m \in E_l$. An element n_m is a name in $[\![i_{ntr.a}, i_{ntr.a.s}]\!]$ if $[i_{ntr.a.s}\ n_m] = i_{ntr.a}$. An element n_m a name in $[\![i_{ntr.a.s}]\!]$ if n_m is a name in $[\![i_{ntr.a}, i_{ntr.a.s}]\!]$ for some $i_{ntr.a}$.

An element d_f of the form $(p_t, (v_{r.*}), (v_{r.s.*}), b_d)$ is an element definition if $(p_t, (v_{r.*}), (v_{r.s.*}))$ is a pattern specification, and $conf :: in \notin \{v_{r.*}\} \cup \{v_{r.s.*}\}$. Let D_f be a set of element definitions.

The objects p_t, $(v_{r.*})$, $(v_{r.s.*})$ and b_d are called a pattern, element variable specification, sequence variable specification and body in $[\![d_f]\!]$. The elements of $v_{r.*}$ and $v_{r.s.*}$ are called element pattern variables and sequence pattern variables in $[\![d_f]\!]$, respectively.

An attribute element $d_{f.s}$ is called an element definition specification if $[support\ d_{f.s}] \subseteq E_l$, and $[image\ d_{f.s}] \subseteq D_f$. A definition d_f is an element definition in $[\![d_{f.s}]\!]$ if $[d_{f.s}\ n_m] = d_f$ for some $n_m \in E_l$. An element n_m is a name in $[\![d_f, d_{f.s}]\!]$ if $[d_{f.s}\ n_m] = d_f$. An element n_m a name in $[\![d_{f.s}]\!]$ if n_m is a name in $[\![d_f, d_{f.s}]\!]$ for some d_f.

Let $[support\ i_{ntr.a.s}] \cap [support\ d_{f.s}] = \emptyset$.

An element $o_{rd.intr}$ of the form $(n_{m.*})$ is called an element interpretation order in $[\![i_{ntr.a.s}, d_{f.s}]\!]$ if $\{n_{m.*}\} \subseteq [support\ i_{ntr.a.s}] \cup [support\ d_{f.s}]$, and the elements of $n_{m.*}$ are pairwise distinct. It specifies the order of application of atomic element interpretations and element definitions to the element to be interpreted.

The information about the element definition specification and element interpretation order of configurations is stored in the substate *interpretation* of the configurations. The conceptuals $(0 : definitions) :: state :: interpretation$ and $(0 : order) :: state :: interpretation$ define the element definition specification and element interpretation order of the configurations, respectively.

An element c_{nf} is consistent with $(i_{ntr.a.s}, d_{f.s}, o_{rd.intr})$ if the following properties hold:

- if $[support\ i_{ntr.a.s}] \cap [support\ [c_{nf}\ (0 : definitions) :: state :: interpretation]] = \emptyset$;
- $d_{f.s} \subseteq [c_{nf}\ (0 : definitions) :: state :: interpretation]$;
- if $n_{m.1} \prec_{[\![o_{rd.intr}]\!]} n_{m.2}$, and $n_{m.1},\ n_{m.2} \in [c_{nf}\ (0 : order) :: state :: interpretation]$, then $n_{m.1} \prec_{[\![[c_{nf}\ (0:order)::state::interpretation]]\!]} n_{m.2}$.

A function $value \in E_l \times C_{nf} \to E_l$ is an element interpretation in $[\![i_{ntr.a.s}, d_{f.s}, o_{rd.intr}, m_t]\!]$ if $[value\ e_l\ c_{nf}] = [value\ e_l\ c_{nf}\ [c_{nf}\ (0 : order) :: state :: interpretation]]$. It specifies interpretation of elements in the context of configurations. The element $[value\ e_l\ c_{nf}]$ is called a value in $[\![e_l, c_{nf}]\!]$.

The auxiliary function $value \in E_l \times C_{nf} \times N_{m.(*)} \to E_l$ is defined by the following rules (the first proper rule is applied):

- if c_{nf} is not consistent with $(i_{ntr.a.s}, d_{f.s}, o_{rd.intr})$, then $[value\ e_l\ c_{nf}\ n_{m.(*)}] = und$;
- if $i_{ntr.a} = [i_{ntr.a.s}\ n_m]$, e_l is an instance in $[\![p_{t.s}[\![i_{ntr.a}]\!], m_t, s_b]\!]$, and $[f_n[\![i_{ntr.a}]\!]\ s_b \cup (conf :: in : c_{nf})] \neq und$, then $[value\ e_l\ c_{nf}\ (n_m\ n_{m.*})] = [f_n[\![i_{ntr.a}]\!]\ [s_b\ conf : c_{nf}]]$;
- if $d_f = [[c_{nf}\ (0 : definitions) :: state :: interpretation]\ n_m]$, e_l is an instance in

$[\![p_{t.s}[\![d_f]\!], m_t, s_b]\!]$, and $[value\ [subst\ s_b \cup (conf\ ::\ in\ :\ c_{nf})\ b_d[\![d_f]\!]]\ c_{nf}] \neq und$, then $[value\ e_l\ c_{nf}\ (n_m\ n_{m.*})] = [value\ [subst\ [s_b\ conf : c_{nf}]\ b_d[\![d_f]\!]]\ c_{nf}]$;

- $[value\ e_l\ c_{nf}\ (n_m\ n_{m.*})] = [value\ e_l\ c_{nf}\ (n_{m.*})]$;
- $[value\ e_l\ c_{nf}\ ()] = und$.

3.9. Satisfiable and valid elements

An element e_l is satisfiable in $[\![(v_{r.*}), c_{nf}]\!]$ if there exists s_b such that $[support\ s_b] = \{v_{r.*}\}$, and $[value\ [subst\ s_b\ e_l]\ c_{nf}] \neq und$.

An element e_l is valid in $[\![(v_{r.*}), c_{nf}]\!]$ if $[value\ [subst\ s_b\ e_l]\ c_{nf}] \neq und$ for each s_b such that $[support\ s_b] = \{v_{r.*}\}$.

3.10. Conceptual configuration systems

An object $s_{s.c.c}$ of the form $(A_{tm}, i_{ntr.a.s}, d_{f.s}, o_{rd.intr}, m_t)$ is called a conceptual configuration system if $i_{ntr.a.s}$, $d_{f.s}$, $o_{rd.intr}$ and m_t are an atomic element interpretation specification, element definition specification element interpretation order and match in $[\![A_{tm}]\!]$, and $[support\ i_{ntr.a.s}] \cap [support\ d_{f.s}] = \emptyset$. Let $S_{s.c.c}$ be a set of conceptual configuration systems.

The elements of A_{tm}, $E_l[\![A_{tm}]\!]$, $C_{ncpl}[\![A_{tm}]\!]$, $S_{tt}[\![A_{tm}]\!]$ and $C_{nf}[\![A_{tm}]\!]$ are called atoms, elements, conceptuals, states and configurations in $[\![s_{s.t.c}]\!]$.

The objects $i_{ntr.a.s}$, $d_{f.s}$, $o_{rd.intr}$ and m_t are called atomic element interpretation specification, element definition specification, element interpretation order and match in $[\![s_{s.c.c}]\!]$.

An element e_l is interpretable in $[\![s_{s.c.c}]\!]$ if there exist n_m such that e_l is an instance in $[\![p_{t.s}[\![[i_{ntr.a.s}\ n_m]]\!], m_t]\!]$, or e_l is an instance in $[\![p_{t.s}[\![[d_{f.s}\ n_m]]\!], m_t]\!]$.

3.11. Conceptual information query models

An object $m_{dl.q.i.c}$ of the form $(s_{s.c.c}, r_{pr.s}, r_{pr.q}, r_{pr.a})$ is a conceptual information query model in $[\![s_{s.q.i}]\!]$ if $r_{pr.s}, r_{pr.q}, r_{pr.a} \in F_n$, $[support\ r_{pr.s}] = O_{b.s}[\![s_{s.q.i}]\!]$, $[image\ r_{pr.s}] \subseteq E_l[\![s_{s.c.c}]\!]$, $[image\ r_{pr.s}\ S_{tt}[\![s_{s.q.i}]\!]] \subseteq C_{nf}[\![s_{s.c.c}]\!]$, $[support\ r_{pr.q}] = O_{b.q}[\![s_{s.q.i}]\!]$, $[image\ r_{pr.q}] \subseteq E_l[\![s_{s.c.c}]\!]$, $[support\ r_{pr.a}] = O_{b.a}[\![s_{s.q.i}]\!]$, $[image\ r_{pr.a}] \subseteq E_l[\![s_{s.c.c}]\!]$, and $[r_{pr.a}\ [value\ q_r\ s_{tt}]] = [value\ [r_{pr.q}\ q_r]\ [r_{pr.s}\ s_{tt}]]$. Let $M_{dl.q.i.c}$ be a set of conceptual information query models.

The system $s_{s.c.c}$ is called a conceptual configuration system in $[\![m_{dl.q.i.c}]\!]$. The functions $r_{pr.s}$, $r_{pr.q}$ and $r_{pr.a}$ are called a state representation, query representation and answer representation in $[\![m_{dl.q.i.c}]\!]$, respectively.

A system $s_{s.q.i}$ is conceptually modelled in $[\![s_{s.c.c}]\!]$ if there exists $m_{dl.q.i.c}$ such that $s_{s.c.c} = s_{s.c.c}[\![m_{dl.q.i.c}]\!]$, and $m_{dl.q.i.c}$ is a conceptual query model in $[\![s_{s.q.i}]\!]$. The set $[image\ r_{pr.s}]$ is called an ontology in $[\![s_{s.q.i}, m_{dl.q.i.c}]\!]$. It includes conceptual structures of $s_{s.c.c}[\![m_{dl.q.i.c}]\!]$ representing the conceptual structure of state objects in $[\![s_{s.q.i}]\!]$.

Let $r^{-}_{pr.s}$, $r^{-}_{pr.q}$ and $r^{-}_{pr.a}$ denote the inverse functions of $r_{pr.s}$, $r_{pr.q}$ and $r_{pr.a}$ in the case of their existence.

3.12. Extensions

A system $s_{s.q.i.1}$ is an extension of $s_{s.q.i.2}$ if $s_t[\![s_{s.q.i.1}]\!] \subseteq s_t[\![s_{s.q.i.2}]\!]$ for each $s_t \in \{S_{tt}, O_{b.s}, Q_r, O_{b.q}, A_{ns}, O_{b.a}, value\}$.

A system $s_{s.c.c.1}$ is an extension of $s_{s.c.c.2}$ if $o_b[\![s_{s.c.c.1}]\!] = o_b[\![s_{s.c.c.2}]\!]$ for each $o_b \in \{A_{tm}, m_t\}$, $s_t[\![s_{s.c.c.1}]\!] \subseteq s_t[\![s_{s.c.c.2}]\!]$ for each $s_t \in \{i_{ntr.a.s}, d_{f.s}\}$, and the following property hold:

- if $n_{m.1} \prec_{[\![o_{rd.intr}[\![s_{s.c.c.1}]\!]]\!]} n_{m.2}$, and $n_{m.1},\ n_{m.2} \in o_{rd.intr}[\![s_{s.c.c.2}]\!]$, then $n_{m.1} \prec_{[\![o_{rd.intr}[\![s_{s.c.c.2}]\!]]\!]} n_{m.2}$.

A CCS l_n is a language of CCSs if the conceptual structures (atoms, elements, conceptuals and so on) of l_n is syntactically defined.

4. Structure of conceptuals

4.1. Elements of conceptuals

An element e_l is an element in $[\![c_{ncpl}, i_{nt}]\!]$ if $e_l = [c_{ncpl}\ i_{nt}]$ and $e_l \neq und$.

⨁ Let $c_{ncpl} = (-3 : 10, -2 : inch, -1 : area, 0 : f_g, 1 : triangle, 2 : Euclidean, 3 : 2)$. Then 10, *inch*, *area*, f_g, *triangle*, *Euclidean*, 2 are elements in $[\![c_{ncpl}]\!]$ in $[\![-3]\!]$, $[\![-2]\!]$, $[\![-1]\!]$, $[\![0]\!]$, $[\![1]\!]$, $[\![2]\!]$, $[\![3]\!]$.

An element e_l is an element in $[\![c_{ncpl}]\!]$ if there exists i_{nt} such that e_l is an element in $[\![c_{ncpl}, i_{nt}]\!]$.

⨁ Let $c_{ncpl} = (-3 : 10, -2 : inch, -1 : area, 0 : f_g, 1 : triangle, 2 : Euclidean, 3 : 2)$. Then 10, *inch*, *area*, f_g, *triangle*, *Euclidean*, 2 are elements in $[\![c_{ncpl}]\!]$.

4.2. Orders of conceptuals in the context of elements

A number i_{nt} is an order in $[\![c_{ncpl}, e_l]\!]$ if $e_l = [c_{ncpl}\ i_{nt}]$ and $e_l \neq und$. Let O_{rd} be a set of objects called orders.

⨁ Let $c_{ncpl} = (-3 : 10, -2 : inch, -1 : area, 0 : f_g, 1 : triangle, 2 : Euclidean, 3 : 2)$. Then -3, -2, -1, 0, 1, 2, 3 are orders in $[\![c_{ncpl}]\!]$ in $[\![10]\!]$, $[\![inch]\!]$, $[\![area]\!]$, $[\![f_g]\!]$, $[\![triangle]\!]$, $[\![Euclidean]\!]$, $[\![3]\!]$.

A number i_{nt} is an order in $[\![c_{ncpl}, element :]\!]$ if there exists e_l such that i_{nt} is an order in $[\![c_{ncpl}, e_l]\!]$.

⨁ Let $c_{ncpl} = (-3 : 10, -2 : inch, -1 : area, 0 : f_g, 1 : triangle, 2 : Euclidean, 3 : 2)$. Then -3, -2, -1, 0, 1, 2, 3 are orders in $[\![c_{ncpl}, element :]\!]$.

4.3. Properties of elements of conceptuals

Proposition 1. The element *und* is not an element in $[\![c_{ncpl}]\!]$.

Proof. This follows from the definition of element in $[\![c_{ncpl}]\!]$. □

Proposition 2. The number of elements in $[\![c_{ncpl}]\!]$ is finite.

Proof. This follows from the fact that $[support\ c_{ncpl}]$ is finite and *und* is not an element in $[\![c_{ncpl}]\!]$. □

4.4. Properties of orders of conceptuals in the context of elements

Proposition 3. The number of orders in $[\![c_{ncpl}, e_l[\![c_{ncpl}]\!]]\!]$ is finite.

Proof. This follows from the fact that $[support\ c_{ncpl}]$ is finite and *und* is not an element in $[\![c_{ncpl}]\!]$. □

Proposition 4. The number of orders in $[\![c_{ncpl}, element :]\!]$ is finite.

Proof. This follows from the fact that $[support\ c_{ncpl}]$ is finite. □

4.5. Kinds of orders of conceptuals in the context of elements

An order $o_{rd}[\![c_{ncpl}, e_l]\!]$ is minimal in $[\![c_{ncpl}, e_l]\!]$ if i_{nt} is not an order in $[\![c_{ncpl}, e_l]\!]$ for each i_{nt} such that $i_{nt} < o_{rd}$.

⨁ Let $c_{ncpl} = (-3 : 10, -2 : inch, -1 : area, 0 : f_g, 1 : triangle, 2 : Euclidean, 3 : 2)$. Then -2 is a minimal order in $[\![c_{ncpl}, inch]\!]$.

An order $o_{rd}[\![c_{ncpl}]\!]$ is minimal in $[\![c_{ncpl}, element :]\!]$ if i_{nt} is not an order in $[\![c_{ncpl}, \hat{e}_l]\!]$ for each i_{nt} such that $i_{nt} < o_{rd}$.

⨁ Let $c_{ncpl} = (-3 : 10, -2 : inch, -1 : area, 0 : f_g, 1 : triangle, 2 : Euclidean, 3 : 2)$. Then -3 is a minimal order in $[\![c_{ncpl}, element :]\!]$.

An order $o_{rd}[\![c_{ncpl}, e_l]\!]$ is maximal in $[\![c_{ncpl}, e_l]\!]$ if i_{nt} is not an order in $[\![c_{ncpl}, e_l]\!]$ for each i_{nt} such that $o_{rd} < i_{nt}$.

$\bigoplus$ Let $c_{ncpl} = (-3 : 10, -2 : inch, -1 : area, 0 : f_g, 1 : triangle, 2 : Euclidean, 3 : 10)$. Then 2 is a maximal order in $[\![c_{ncpl}, Euclidean]\!]$.

An order $o_{rd}[\![c_{ncpl}]\!]$ is maximal in $[\![c_{ncpl}, element :]\!]$ if i_{nt} is not an order in $[\![c_{ncpl}, \hat{e}_l]\!]$ for each i_{nt} such that $o_{rd} < i_{nt}$.

$\bigoplus$ Let $c_{ncpl} = (-3 : 10, -2 : inch, -1 : area, 0 : f_g, 1 : triangle, 2 : Euclidean, 3 : 2)$. Then 3 is a maximal order in $[\![c_{ncpl}, element :]\!]$.

4.6. Kinds of elements of conceptuals

An element e_l is minimal in $[\![c_{ncpl}]\!]$ if there exists $o_{rd}[\![c_{ncpl}, e_l]\!]$ such that o_{rd} is minimal in $[\![c_{ncpl}, element :]\!]$.

$\bigoplus$ Let $c_{ncpl} = (-3 : 10, -2 : inch, -1 : area, 0 : f_g, 1 : triangle, 2 : Euclidean, 3 : 2)$. Then 10 is a minimal element in $[\![c_{ncpl}]\!]$.

An element e_l is maximal in $[\![c_{ncpl}]\!]$ if there exists $o_{rd}[\![c_{ncpl}, e_l]\!]$ such that o_{rd} is a maximal order in $[\![c_{ncpl}, element :]\!]$.

$\bigoplus$ Let $c_{ncpl} = (-3 : 10, -2 : inch, -1 : area, 0 : f_g, 1 : triangle, 2 : Euclidean, 3 : 2)$. Then 2 is a maximal element in $[\![c_{ncpl}]\!]$.

An element e_l is null in $[\![c_{ncpl}]\!]$ if e_l is an element in $[\![c_{ncpl}, 0]\!]$.

$\bigoplus$ Let $c_{ncpl} = (-3 : 10, -2 : inch, -1 : area, 0 : f_g, 1 : triangle, 2 : Euclidean, 3 : 2)$. Then f_g is null in $[\![c_{ncpl}]\!]$.

5. Structure of conceptual states

5.1. Conceptuals

A conceptual c_{ncpl} is a conceptual in $[\![s_{tt}]\!]$ if $[value\ c_{ncpl}\ s_{tt}] \neq und$.

A conceptual $c_{ncpl.n}$ is a conceptual in $[\![c_{nf}]\!]$ if $c_{ncpl}[\![c_{ncpl.n}]\!]$ is a conceptual in $[\![[c_{nf}\ n_m\ [\![c_{ncpl.n}]\!]]\!]]$. A conceptual c_{ncpl} is a conceptual in $[\![c_{nf}]\!]$ if there exists n_m such that $c_{ncpl} :: state :: n_m$ is a conceptual in $[\![c_{nf}]\!]$.

$\bigoplus$ Let $c_{ncpl} = (-3 : 10, -2 : inch, -1 : area, 0 : f_g, 1 : triangle, 2 : Euclidean, 3 : 2)$ and $[support\ s_{tt}] = \{c_{ncpl}\}$. Then c_{ncpl} is a conceptual in $[\![s_{tt}]\!]$.

5.2. Elements, orders, concretizations

An element e_l is an element in $[\![s_{tt}, i_{nt}, c_{ncpl}]\!]$ if c_{ncpl} is a conceptual in $[\![s_{tt}]\!]$ and e_l is an element in $[\![c_{ncpl}, i_{nt}]\!]$. An element e_l is an element in $[\![c_{nf}, i_{nt}, c_{ncpl.n}]\!]$ if e_l is an element in $[\![[c_{nf}\ [name\ in\ c_{ncpl.n}]], i_{nt}, [conceptual\ in\ c_{ncpl.n}]]\!]$.

A number i_{nt} is an order in $[\![e_l, s_{tt}, c_{ncpl}]\!]$ if e_l is an element in $[\![s_{tt}, i_{nt}, c_{ncpl}]\!]$. A number i_{nt} is an order in $[\![e_l, c_{nf}, c_{ncpl.n}]\!]$ if e_l is an element in $[\![c_{nf}, i_{nt}, c_{ncpl.n}]\!]$.

A conceptual c_{ncpl} is a concretization in $[\![e_l, s_{tt}, i_{nt}]\!]$ if e_l is an element in $[\![s_{tt}, i_{nt}, c_{ncpl}]\!]$. A conceptual $c_{ncpl.n}$ is a concretization in $[\![e_l, c_{nf}, i_{nt}]\!]$ if e_l is an element in $[\![c_{nf}, i_{nt}, c_{ncpl.n}]\!]$.

5.3. Kinds of elements

An element e_l is an element in $[\![s_{tt}, i_{nt}]\!]$ if there exists c_{ncpl} such that e_l is an element in $[\![s_{tt}, i_{nt}, c_{ncpl}]\!]$. An element e_l is an element in $[\![c_{nf}, i_{nt}]\!]$ if there exists $c_{ncpl.n}$ such that e_l is an element in $[\![c_{nf}, i_{nt}, c_{ncpl.n}]\!]$.

An element e_l is an element in $[\![s_{tt}, c_{ncpl}]\!]$ if there exists i_{nt} such that e_l is an element in $[\![s_{tt}, i_{nt}, c_{ncpl}]\!]$. An element e_l is an element in $[\![c_{nf}, c_{ncpl.n}]\!]$ if there exists i_{nt} such that e_l is an element in $[\![c_{nf}, i_{nt}, c_{ncpl.n}]\!]$.

An element e_l is an element in $[\![s_{tt}]\!]$ if there exists i_{nt} such that e_l is an element in $[\![s_{tt}, i_{nt}]\!]$. An element e_l is an element in $[\![c_{nf}]\!]$ if there exists i_{nt} such that e_l is an element in $[\![c_{nf}, i_{nt}]\!]$.

5.4. Kinds of orders

A number i_{nt} is an order in $[\![e_l, s_{tt}]\!]$ if e_l is an element in $[\![s_{tt}, i_{nt}]\!]$. A number i_{nt} is an order in $[\![e_l, c_{nf}]\!]$ if e_l is an element in $[\![c_{nf}, i_{nt}]\!]$.

A number i_{nt} is an order in $[\![s_{tt}, element:]\!]$ if there exists e_l such that i_{nt} is an order in $[\![e_l, s_{tt}]\!]$. A number i_{nt} is an order in $[\![c_{nf}, element:]\!]$ if there exists e_l such that i_{nt} is an order in $[\![e_l, c_{nf}]\!]$.

5.5. Kinds of concretizations

A conceptual c_{ncpl} is a concretization in $[\![e_l, s_{tt}]\!]$ if e_l is an element in $[\![s_{tt}, c_{ncpl}]\!]$. A conceptual $c_{ncpl.n}$ is a concretization in $[\![e_l, c_{nf}]\!]$ if e_l is an element in $[\![c_{nf}, c_{ncpl.n}]\!]$.

A conceptual c_{ncpl} is a concretization in $[\![s_{tt}, element:]\!]$ if there exists e_l such that c_{ncpl} is a concretization in $[\![e_l, s_{tt}]\!]$. A conceptual $c_{ncpl.n}$ is a concretization in $[\![c_{nf}, element:]\!]$ if there exists e_l such that $c_{ncpl.n}$ is a concretization in $[\![e_l, c_{nf}]\!]$.

5.6. Example

$\bigoplus$ Let $c_{ncl.1} = (-3 : 10, -2 : inch, -1 : area, 0 : e_{l.g.1}, 1 : triangle, 2 : Euclidean, 3 : 2)$, $c_{ncl.2} = (-3 : 8, -2 : cm, -1 : volume, 0 : e_{l.g.2}, 1 : cube, 2 : Lobachevskian, 3 : 3)$, and $[support\ s_{tt}] = \{c_{ncl.1}, c_{ncl.2}\}$. Then the following properties hold:

- 10, 8, *inch*, *cm*, *area*, *volume*, $e_{l.g.1}$, $e_{l.g.2}$, *trianle*, *cube*, *Euclidean*, *Lobachevskian*, 3, 2 are elements in $[\![s_{tt}]\!]$;
- −3, −2, −1, 0, 1, 2, 3 are orders in $[\![s_{tt}, element :]\!]$;
- $c_{ncl.1}$, $c_{ncl.2}$ are concretizations in $[\![s_{tt}, element :]\!]$.

5.7. Properties of elements

Proposition 5. For all e_l and i_{nt} there exist s_{tt} and c_{ncpl} such that e_l is an element in $[\![s_{tt}, i_{nt}, c_{ncpl}]\!]$.

Proof. We define s_{tt} and c_{ncpl} as follows: $[c_{ncpl}\ i_{nt}] = e_l$ and $[s_{tt}\ c_{ncpl}] \neq und$. Then e_l is an element in $[\![s_{tt}, i_{nt}, c_{ncpl}]\!]$. □

6. Classification of elements of states

Elements in $[\![s_{tt}]\!]$ are subclassified into individuals, concepts and attributes.

6.1. Individuals

Individuals in $[\![s_{tt}]\!]$ model elements in $[\![s_{s.q.i}]\!]$.

An element e_l is an individual in $[\![s_{tt}, c_{ncpl}]\!]$ if e_l is an element in $[\![s_{tt}, 0, c_{ncpl}]\!]$. An element e_l is an individual in $[\![c_{nf}, c_{ncpl.n}]\!]$ if e_l is an element in $[\![c_{nf}, 0, c_{ncpl.n}]\!]$.

$\bigoplus$ Let $c_{ncpl} = (-3 : 10, -2 : inch, -1 : area, 0 : f_g, 1 : triangle, 2 : Euclidean, 3 : 2)$ and $s_{tt} = (c_{ncpl} : 3)$. Then f_g is an individual in $[\![s_{tt}, c_{ncpl}]\!]$.

An element e_l is an individual in $[\![s_{tt}]\!]$ if there exists c_{ncpl} such that e_l is an individual in $[\![s_{tt}, c_{ncpl}]\!]$. An element e_l is an individual in $[\![c_{nf}]\!]$ if there exists $c_{ncpl.n}$ such that e_l is an individual in $[\![c_{nf}, c_{ncpl.n}]\!]$.

$\bigoplus$ Let $c_{ncl.1} = (-3 : 10, -2 : inch, -1 : area, 0 : e_{l.g.1}, 1 : triangle, 2 : Euclidean, 3 : 2)$, $c_{ncl.2} = (-3 : 8, -2 : cm, -1 : volume, 0 : e_{l.g.2}, 1 : cube, 2 : Lobachevskian, 3 : 3)$, and $s_{tt} = (c_{ncl.1} : 3, c_{ncl.2} : 4)$. Then $e_{l.g.1}$ and $e_{l.g.2}$ are individuals in $[\![s_{tt}]\!]$.

6.2. Concepts

Concepts in $[\![s_{tt}]\!]$ generalize models of the usual concepts in $[\![s_{s.q.i}]\!]$ which are interpreted as sets of elements in $[\![s_{s.q.i}]\!]$.

An element e_l is a concept in $[\![s_{tt}, n_t, c_{ncpl}]\!]$ if e_l is an element in $[\![s_{tt}, n_t, c_{ncpl}]\!]$. A number n_t is an order in $[\![e_l, s_{tt}, c_{ncpl}]\!]$ in $[\![concept : e_l, s_{tt}, c_{ncpl}]\!]$ if e_l is a concept in $[\![s_{tt}, n_t, c_{ncpl}]\!]$. A conceptual c_{ncpl} is a concretization in $[\![concept : e_l, s_{tt}, n_t]\!]$ if e_l is a concept in $[\![s_{tt}, n_t, c_{ncpl}]\!]$.

An element e_l is a concept in $[\![c_{nf}, n_t, c_{ncpl.n}]\!]$ if e_l is an element in $[\![c_{nf}, n_t, c_{ncpl.n}]\!]$. A number n_t is an order in $[\![e_l, c_{nf}, c_{ncpl.n}]\!]$ in $[\![concept : e_l, c_{nf}, c_{ncpl.n}]\!]$ if e_l is a concept in $[\![c_{nf}, n_t, c_{ncpl.n}]\!]$. A conceptual $c_{ncpl.n}$ is a concretization in $[\![concept : e_l, c_{nf}, n_t]\!]$ if e_l is a concept in $[\![c_{nf}, n_t, c_{ncpl.n}]\!]$.

⊕ Let $c_{ncpl} = (-3 : 10, -2 : inch, -1 : area, 0 : f_g, 1 : triangle, 2 : Euclidean, 3 : 2)$ and $s_{tt} = (c_{ncpl} : 3)$. Then the following properties hold:

- $triangle$, $Euclidean$, 2 are concepts in $[\![s_{tt}]\!]$ in $[\![1]\!]$, $[\![2]\!]$, $[\![3]\!]$ in $[\![c_{ncpl}]\!]$;
- 1, 2, 3 are orders in $[\![concept : triangle]\!]$, $[\![concept : Euclidean]\!]$, $[\![concept : 2]\!]$ in $[\![s_{tt}]\!]$ in $[\![c_{ncpl}]\!]$;
- c_{ncpl} is a concretization in $[\![concept : triangle]\!]$, $[\![concept : Euclidean]\!]$, $[\![concept : 3]\!]$ in $[\![s_{tt}]\!]$ in $[\![1]\!]$, $[\![2]\!]$, $[\![2]\!]$.

An element e_l is a concept in $[\![s_{tt}, n_t]\!]$ if there exists c_{ncpl} such that e_l is a concept in $[\![s_{tt}, n_t, c_{ncpl}]\!]$. A number n_t is an order in $[\![e_l, s_{tt}]\!]$ in $[\![concept : e_l, s_{tt}]\!]$ if e_l is a concept in $[\![s_{tt}, n_t]\!]$.

An element e_l is a concept in $[\![c_{nf}, n_t]\!]$ if there exists $c_{ncpl.n}$ such that e_l is a concept in $[\![c_{nf}, n_t, c_{ncpl.n}]\!]$. A number n_t is an order in $[\![e_l, c_{nf}]\!]$ in $[\![concept : e_l, c_{nf}]\!]$ if e_l is a concept in $[\![c_{nf}, n_t]\!]$.

⊕ Let $c_{ncl.1} = (-3 : 10, -2 : inch, -1 : area, 0 : e_{l.g.1}, 1 : triangle, 2 : Euclidean, 3 : 2)$, $c_{ncl.2} = (-3 : 8, -2 : cm, -1 : volume, 0 : e_{l.g.2}, 1 : cube, 2 : Lobachevskian, 3 : 3)$, and $s_{tt} = (c_{ncl.1} : 3, c_{ncl.2} : 4)$. Then the following properties hold:

- $triangle$, $Euclidean$, 2 are concepts in $[\![s_{tt}]\!]$ in $[\![1]\!]$, $[\![2]\!]$, $[\![2]\!]$;
- $cube$, $Lobachevskian$, 3 are concepts in $[\![s_{tt}]\!]$ in $[\![1]\!]$, $[\![2]\!]$, $[\![3]\!]$;
- 1, 2, 3 are orders in $[\![concept : triangle]\!]$, $[\![concept : Euclidean]\!]$, $[\![concept : 2]\!]$ in $[\![s_{tt}]\!]$;
- 1, 2, 3 are orders in $[\![concept : cube]\!]$, $[\![concept : Lobachevskian]\!]$, $[\![concept : 3]\!]$ in $[\![s_{tt}]\!]$.

An element e_l is a concept in $[\![s_{tt}, c_{ncpl}]\!]$ if there exists n_t such that e_l is a concept in $[\![s_{tt}, n_t, c_{ncpl}]\!]$. A conceptual c_{ncpl} is a concretization in $[\![e_l, s_{tt}]\!]$ in $[\![concept : e_l, s_{tt}]\!]$ if e_l is a concept in $[\![s_{tt}, c_{ncpl}]\!]$.

An element e_l is a concept in $[\![c_{nf}, c_{ncpl.n}]\!]$ if there exists n_t such that e_l is a concept in

$[\![c_{nf}, n_t, c_{ncpl.n}]\!]$. A conceptual $c_{ncpl.n}$ is a concretization in $[\![e_l, c_{nf}]\!]$ in $[\![concept : e_l, c_{nf}]\!]$ if e_l is a concept in $[\![c_{nf}, c_{ncpl.n}]\!]$.

⨁ Let $c_{ncl.1} = (-3 : 10, -2 : inch, -1 : area, 0 : e_{l.g.1}, 1 : triangle, 2 : Euclidean, 3 : 2)$, $c_{ncl.2} = (-3 : 8, -2 : cm, -1 : volume, 0 : e_{l.g.2}, 1 : cube, 2 : Lobachevskian, 3 : 3)$, and $s_{tt} = (c_{ncl.1} : 3, c_{ncl.2} : 4)$. Then the following properties hold:

– *triangle*, *Euclidean*, 2 are concepts in $[\![s_{tt}, c_{ncl.1}]\!]$;

– *cube*, *Lobachevskian*, 3 are concepts in $[\![s_{tt}, c_{ncl.2}]\!]$;

– $c_{ncl.1}$ is a concretization in $[\![concept : triangle]\!]$, $[\![concept : Euclidean]\!]$, $[\![concept : 2]\!]$ in $[\![s_{tt}]\!]$;

– $c_{ncl.2}$ is a concretization in $[\![concept : cube]\!]$, $[\![concept : Lobachevskian]\!]$, $[\![concept : 3]\!]$ in $[\![s_{tt}]\!]$.

An element e_l is a concept in $[\![s_{tt}]\!]$ if there exists n_t such that e_l is a concept in $[\![s_{tt}, n_t]\!]$. A number n_t is an order in $[\![s_{tt}]\!]$ in $[\![s_{tt}, concept :]\!]$ if there exists e_l such that n_t is an order in $[\![concept : e_l, s_{tt}]\!]$. A conceptual c_{ncpl} is a concretization in $[\![s_{tt}]\!]$ in $[\![s_{tt}, concept :]\!]$ if there exists e_l such that c_{ncpl} is a concretization in $[\![concept : e_l, s_{tt}]\!]$.

An element e_l is a concept in $[\![c_{nf}]\!]$ if there exists n_t such that e_l is a concept in $[\![c_{nf}, n_t]\!]$. A number n_t is an order in $[\![c_{nf}]\!]$ in $[\![c_{nf}, concept :]\!]$ if there exists e_l such that n_t is an order in $[\![concept : e_l, c_{nf}]\!]$. A conceptual $c_{ncpl.n}$ is a concretization in $[\![c_{nf}]\!]$ in $[\![c_{nf}, concept :]\!]$ if there exists e_l such that $c_{ncpl.n}$ is a concretization in $[\![concept : e_l, c_{nf}]\!]$.

⨁ Let $c_{ncl.1} = (-3 : 10, -2 : inch, -1 : area, 0 : e_{l.g.1}, 1 : triangle, 2 : Euclidean, 3 : 2)$, $c_{ncl.2} = (-3 : 8, -2 : cm, -1 : volume, 0 : e_{l.g.2}, 1 : cube, 2 : Lobachevskian, 3 : 3)$, and $s_{tt} = (c_{ncl.1} : 3, c_{ncl.2} : 4)$. Then the following properties hold:

– *triangle*, *Euclidean*, 2, *cube*, *Lobachevskian*, 3 are concepts in $[\![s_{tt}]\!]$;

– 1, 2, 3 are orders in $[\![s_{tt}, concept :]\!]$;

– $c_{ncl.1}$, $c_{ncl.2}$ are concretizations in $[\![s_{tt}, concept :]\!]$.

6.3. Attributes

Attributes in $[\![s_{tt}]\!]$ generalize models of the usual attributes in $[\![s_{s.q.i}]\!]$ which are interpreted as characteristics of elements of $s_{s.q.i}$.

An element e_l is an attribute in $[\![s_{tt}, n_t, c_{ncpl}]\!]$ if e_l is an element in $[\![s_{tt}, -n_t, c_{ncpl}]\!]$. A number n_t is an order in $[\![e_l, s_{tt}, c_{ncpl}]\!]$ in $[\![attribute : e_l, s_{tt}, c_{ncpl}]\!]$ if e_l is an attribute in $[\![s_{tt}, n_t, c_{ncpl}]\!]$. A conceptual c_{ncpl} is a concretization in $[\![attribute : e_l, s_{tt}, n_t]\!]$ if e_l is an attribute in $[\![s_{tt}, n_t, c_{ncpl}]\!]$.

An element e_l is an attribute in $[\![c_{nf}, n_t, c_{ncpl.n}]\!]$ if e_l is an element in $[\![c_{nf}, -n_t, c_{ncpl.n}]\!]$. A number n_t is an order in $[\![e_l, c_{nf}, c_{ncpl.n}]\!]$ in $[\![attribute : e_l, c_{nf}, c_{ncpl.n}]\!]$ if e_l is an attribute in $[\![c_{nf}, n_t, c_{ncpl.n}]\!]$. A conceptual $c_{ncpl.n}$ is a concretization in $[\![attribute : e_l, c_{nf}, n_t]\!]$ if e_l is an attribute in $[\![c_{nf}, n_t, c_{ncpl.n}]\!]$.

⨁ Let $c_{ncpl} = (-3 : 10, -2 : inch, -1 : area, 0 : f_g, 1 : triangle, 2 : Euclidean, 3 : 2)$ and $s_{tt} = (c_{ncpl} : 3)$. Then the following properties hold:

- $area$, $inch$, 10 are attributes in $[\![s_{tt}]\!]$ in $[\![1]\!]$, $[\![2]\!]$, $[\![3]\!]$ in $[\![c_{ncpl}]\!]$;
- 1, 2, 3 are orders in $[\![attribute : area]\!]$, $[\![attribute : inch]\!]$, $[\![attribute : 10]\!]$ in $[\![s_{tt}]\!]$ in $[\![c_{ncpl}]\!]$;
- c_{ncpl} is a concretization in $[\![attribute : area]\!]$, $[\![attribute : inch]\!]$, $[\![attribute : 10]\!]$ in $[\![s_{tt}]\!]$ in $[\![1]\!]$, $[\![2]\!]$, $[\![3]\!]$.

An element e_l is an attribute in $[\![s_{tt}, n_t]\!]$ if there exists c_{ncpl} such that e_l is an attribute in $[\![s_{tt}, n_t, c_{ncpl}]\!]$. A number n_t is an order in $[\![e_l, s_{tt}]\!]$ in $[\![attribute : e_l, s_{tt}]\!]$ if e_l is an attribute in $[\![s_{tt}, n_t]\!]$.

An element e_l is an attribute in $[\![c_{nf}, n_t]\!]$ if there exists $c_{ncpl.n}$ such that e_l is an attribute in $[\![c_{nf}, n_t, c_{ncpl.n}]\!]$. A number n_t is an order in $[\![e_l, c_{nf}]\!]$ in $[\![attribute : e_l, c_{nf}]\!]$ if e_l is an attribute in $[\![c_{nf}, n_t]\!]$.

⨁ Let $c_{ncl.1} = (-3 : 10, -2 : inch, -1 : area, 0 : e_{l.g.1}, 1 : triangle, 2 : Euclidean, 3 : 2)$, $c_{ncl.2} = (-3 : 8, -2 : cm, -1 : volume, 0 : e_{l.g.2}, 1 : cube, 2 : Lobachevskian, 3 : 3)$, and $s_{tt} = (c_{ncl.1} : 3, c_{ncl.2} : 4)$. Then the following properties hold:

- $area$, $inch$, 10 are attributes in $[\![s_{tt}]\!]$ in $[\![1]\!]$, $[\![2]\!]$, $[\![3]\!]$;
- $volume$, cm, 8 are attributes in $[\![s_{tt}]\!]$ in $[\![1]\!]$, $[\![2]\!]$, $[\![3]\!]$;
- 1, 2, 3 are orders in $[\![attribute : area]\!]$, $[\![attribute : inch]\!]$, $[\![attribute : 10]\!]$ in $[\![s_{tt}]\!]$;
- 1, 2, 3 are orders in $[\![attribute : volume]\!]$, $[\![attribute : cm]\!]$, $[\![attribute : 8]\!]$ in $[\![s_{tt}]\!]$.

An element e_l is an attribute in $[\![s_{tt}, c_{ncpl}]\!]$ if there exists n_t such that e_l is an attribute in $[\![s_{tt}, n_t, c_{ncpl}]\!]$. A conceptual c_{ncpl} is a concretization in $[\![e_l, s_{tt}]\!]$ in $[\![attribute : e_l, s_{tt}]\!]$ if e_l is an attribute in $[\![s_{tt}, c_{ncpl}]\!]$.

An element e_l is an attribute in $[\![c_{nf}, c_{ncpl.n}]\!]$ if there exists n_t such that e_l is an attribute in $[\![c_{nf}, n_t, c_{ncpl.n}]\!]$. A conceptual $c_{ncpl.n}$ is a concretization in $[\![e_l, c_{nf}]\!]$ in $[\![attribute : e_l, c_{nf}]\!]$ if e_l is an attribute in $[\![c_{nf}, c_{ncpl.n}]\!]$.

⨁ Let $c_{ncl.1} = (-3 : 10, -2 : inch, -1 : area, 0 : e_{l.g.1}, 1 : triangle, 2 : Euclidean, 3 : 2)$, $c_{ncl.2} = (-3 : 8, -2 : cm, -1 : volume, 0 : e_{l.g.2}, 1 : cube, 2 : Lobachevskian, 3 : 3)$, and

$s_{tt} = (c_{ncl.1} : 3, c_{ncl.2} : 4)$. Then the following properties hold:

- $area$, $inch$, 10 are attributes in $[\![s_{tt}, c_{ncl.1}]\!]$;
- $volume$, cm, 8 are attributes in $[\![s_{tt}, c_{ncl.2}]\!]$;
- $c_{ncl.1}$ is a concretization in $[\![attribute : area]\!]$, $[\![attribute : inch]\!]$, $[\![attribute : 10]\!]$ in $[\![s_{tt}]\!]$;
- $c_{ncl.2}$ is a concretization in $[\![attribute : volume]\!]$, $[\![attribute : cm]\!]$, $[\![attribute : 8]\!]$ in $[\![s_{tt}]\!]$.

An element e_l is an attribute in $[\![s_{tt}]\!]$ if there exists n_t such that e_l is an attribute in $[\![s_{tt}, n_t]\!]$. A number n_t is an order in $[\![s_{tt}, attribute :]\!]$ if there exists e_l such that n_t is an order in $[\![attribute : e_l, s_{tt}]\!]$. A conceptual c_{ncpl} is a concretization in $[\![s_{tt}, attribute :]\!]$ if there exists e_l such that c_{ncpl} is a concretization in $[\![attribute : e_l, s_{tt}]\!]$.

An element e_l is an attribute in $[\![c_{nf}]\!]$ if there exists n_t such that e_l is an attribute in $[\![c_{nf}, n_t]\!]$. A number n_t is an order in $[\![c_{nf}, attribute :]\!]$ if there exists e_l such that n_t is an order in $[\![attribute : e_l, c_{nf}]\!]$. A conceptual $c_{ncpl.n}$ is a concretization in $[\![c_{nf}, attribute :]\!]$ if there exists e_l such that $c_{ncpl.n}$ is a concretization in $[\![attribute : e_l, c_{nf}]\!]$.

$\bigoplus$ Let $c_{ncl.1} = (-3 : 10, -2 : inch, -1 : area, 0 : e_{l.g.1}, 1 : triangle, 2 : Euclidean, 3 : 2)$, $c_{ncl.2} = (-3 : 8, -2 : cm, -1 : volume, 0 : e_{l.g.2}, 1 : cube, 2 : Lobachevskian, 3 : 3)$, and $s_{tt} = (c_{ncl.1} : 3, c_{ncl.2} : 4)$. Then the following properties hold:

- $area$, $inch$, 10, $volume$, cm, 8 are attributes in $[\![s_{tt}]\!]$;
- 1, 2, 3 are orders in $[\![s_{tt}, attribute :]\!]$;
- $c_{ncl.1}$, $c_{ncl.2}$ are concretizations in $[\![s_{tt}, attribute :]\!]$.

Concepts and attributes are considered in detail below.

7. Structure of concepts

7.1. Direct concepts

The usual concepts in $[\![s_{s.q.i}]\!]$ which are interpreted as sets of elements in $[\![s_{s.q.i}]\!]$ are modelled by the special kind of concepts in $[\![s_{tt}]\!]$, direct concepts in $[\![s_{tt}]\!]$.

7.1.1. Direct concepts

An element e_l is a direct concept in $[\![s_{tt}, c_{ncpl}]\!]$ if e_l is a concept in $[\![s_{tt}, 1, c_{ncpl}]\!]$. An element e_l is a direct concept in $[\![c_{nf}, c_{ncpl.n}]\!]$ if e_l is a concept in $[\![c_{nf}, 1, c_{ncpl.n}]\!]$.

An element e_l is a direct concept in $[\![s_{tt}]\!]$ if there exists c_{ncpl} such that e_l is a direct concept in $[\![s_{tt}, c_{ncpl}]\!]$. An element e_l is a direct concept in $[\![c_{nf}]\!]$ if there exists $c_{ncpl.n}$ such that e_l is a direct concept in $[\![c_{nf}, c_{ncpl.n}]\!]$.

7.1.2. Concretizations

A conceptual c_{ncpl} is a concretization in $[\![\mathit{direct{-}concept} : e_l, s_{tt}]\!]$ if e_l is a concept in $[\![s_{tt}, 1, c_{ncpl}]\!]$. A conceptual $c_{ncpl.n}$ is a concretization in $[\![\mathit{direct{-}concept} : e_l, c_{nf}]\!]$ if e_l is a concept in $[\![c_{nf}, 1, c_{ncpl.n}]\!]$.

A conceptual c_{ncpl} is a concretization in $[\![s_{tt}, \mathit{direct{-}concept} :]\!]$ if there exists e_l such that c_{ncpl} is a concretization in $[\![\mathit{direct{-}concept} : e_l, s_{tt}]\!]$. A conceptual $c_{ncpl.n}$ is a concretization in $[\![c_{nf}, \mathit{direct{-}concept} :]\!]$ if there exists e_l such that $c_{ncpl.n}$ is a concretization in $[\![\mathit{direct{-}concept} : e_l, c_{nf}]\!]$.

⊕ Let $c_{ncl.1} = (-3 : 10, -2 : \mathit{inch}, -1 : \mathit{area}, 0 : f_g, 1 : \mathit{triangle}, 2 : \mathit{Euclidean}, 3 : 2)$, $c_{ncl.2} = (-3 : 10, -2 : \mathit{inch}, -1 : \mathit{perimeter}, 0 : f_g, 1 : \mathit{rectangle}, 2 : \mathit{Euclidean}, 3 : 2)$, and $[\mathit{support}\ s_{tt}] = \{c_{ncl.1}, c_{ncl.2}\}$. Then the following properties hold:

- $\mathit{triangle}$ and $\mathit{rectangle}$ are direct concepts in s_{tt};
- $c_{ncl.1}$ is a concretization in $[\![\mathit{direct{-}concept} : \mathit{triangle}, s_{tt}]\!]$;
- $c_{ncl.2}$ is a concretization in $[\![\mathit{direct{-}concept} : \mathit{rectangle}, s_{tt}]\!]$.

7.2. Elements of concepts

7.2.1. Elements, orders, concretizations

An element e_l is an element in $[\![\mathit{concept} : c_{ncp}, s_{tt}, \mathit{concept{-}order} : n_t, \mathit{element{-}order} : i_{nt}, c_{ncpl}]\!]$ if c_{ncp} is a concept in $[\![s_{tt}, n_t, c_{ncpl}]\!]$, e_l is an element in $[\![c_{ncpl}, i_{nt}]\!]$, and $i_{nt} < n_t$. An element e_l is an element in $[\![\mathit{concept} : c_{ncp}, c_{nf}, \mathit{concept{-}order} : n_t, \mathit{element{-}order} : i_{nt}, c_{ncpl.n}]\!]$ if c_{ncp} is a concept in $[\![c_{nf}, n_t, c_{ncpl.n}]\!]$, e_l is an element in $[\![c_{ncpl.n}, i_{nt}]\!]$, and $i_{nt} < n_t$.

Thus, elements of c_{ncp} can be concepts of orders which are less than the order of c_{ncp}, individuals and attributes of any orders.

A number n_t is an order in $[\![e_l, \mathit{concept} : c_{ncp}, s_{tt}, \mathit{element{-}order} : i_{nt}, c_{ncpl}]\!]$ if e_l is an element in $[\![\mathit{concept} : c_{ncp}, s_{tt}, \mathit{concept{-}order} : n_t, \mathit{element{-}order} : i_{nt}, c_{ncpl}]\!]$. It specifies the order in $[\![c_{ncpl}, c_{ncp}]\!]$. A number n_t is an order in $[\![e_l, \mathit{concept} : c_{ncp}, c_{nf}, \mathit{element{-}order} : i_{nt}, c_{ncpl.n}]\!]$ if e_l is an element in $[\![\mathit{concept} : c_{ncp}, c_{nf}, \mathit{concept{-}order} : n_t, \mathit{element{-}order} : i_{nt}, c_{ncpl.n}]\!]$.

A number i_{nt} is an order in $[\![e_l, \mathit{concept} : c_{ncp}, s_{tt}, \mathit{concept{-}order} : n_t, c_{ncpl}]\!]$ if e_l is an element in $[\![concept : c_{ncp}, s_{tt}, concept{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}]\!]$. It specifies the order in $[\![c_{ncpl}, e_l]\!]$. A number i_{nt} is an order in $[\![e_l, concept : c_{ncp}, c_{nf}, concept{-}order : n_t, c_{ncpl.n}]\!]$ if e_l is an element in $[\![concept : c_{ncp}, c_{nf}, concept{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}]\!]$.

A conceptual c_{ncpl} is a concretization in $[\![e_l, concept : c_{ncp}, s_{tt}, concept{-}order : n_t, element{-}order : i_{nt}]\!]$ if e_l is an element in $[\![concept : c_{ncp}, s_{tt}, concept{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}]\!]$. It defines that e_l is an element in $[\![concept : c_{ncp}, s_{tt}, concept{-}order : n_t, element{-}order : i_{nt}]\!]$. A conceptual $c_{ncpl.n}$ is a concretization in $[\![e_l, concept : c_{ncp}, c_{nf}, concept{-}order : n_t, element{-}order : i_{nt}]\!]$ if e_l is an element in $[\![concept : c_{ncp}, c_{nf}, concept{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}]\!]$.

7.2.2. Kinds of elements

An element e_l is an element in $[\![concept : c_{ncp}, s_{tt}, concept{-}order : n_t, element{-}order : i_{nt}]\!]$ if there exists c_{ncpl} such that e_l is an element in $[\![concept : c_{ncp}, s_{tt}, concept{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}]\!]$. An element e_l is an element in $[\![concept : c_{ncp}, c_{nf}, concept{-}order : n_t, element{-}order : i_{nt}]\!]$ if there exists $c_{ncpl.n}$ such that e_l is an element in $[\![concept : c_{ncp}, c_{nf}, concept{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}]\!]$.

An element e_l is an element in $[\![concept : c_{ncp}, s_{tt}, concept{-}order : n_t, c_{ncpl}]\!]$ if there exists i_{nt} such that e_l is an element in $[\![concept : c_{ncp}, s_{tt}, concept{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}]\!]$. An element e_l is an element in $[\![concept : c_{ncp}, c_{nf}, concept{-}order : n_t, c_{ncpl.n}]\!]$ if there exists i_{nt} such that e_l is an element in $[\![concept : c_{ncp}, c_{nf}, concept{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}]\!]$.

An element e_l is an element in $[\![concept : c_{ncp}, s_{tt}, element{-}order : i_{nt}, c_{ncpl}]\!]$ if there exists n_t such that e_l is an element in $[\![concept : c_{ncp}, s_{tt}, concept{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}]\!]$. An element e_l is an element in $[\![concept : c_{ncp}, c_{nf}, element{-}order : i_{nt}, c_{ncpl.n}]\!]$ if there exists n_t such that e_l is an element in $[\![concept : c_{ncp}, c_{nf}, concept{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}]\!]$.

An element e_l is an element in $[\![concept : c_{ncp}, s_{tt}, concept{-}order : n_t]\!]$ if there exist i_{nt} and c_{ncpl} such that e_l is an element in $[\![concept : c_{ncp}, s_{tt}, concept{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}]\!]$. An element e_l is an element in $[\![concept : c_{ncp}, c_{nf}, concept{-}order : n_t]\!]$ if there exist i_{nt} and $c_{ncpl.n}$ such that e_l is an element in $[\![concept : c_{ncp}, c_{nf}, concept{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}]\!]$.

An element e_l is an element in $[\![concept : c_{ncp}, s_{tt}, element{-}order : i_{nt}]\!]$ if there exist n_t and

c_{ncpl} such that e_l is an element in $[\![concept : c_{ncp}, s_{tt}, concept{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}]\!]$. An element e_l is an element in $[\![concept : c_{ncp}, c_{nf}, element{-}order : i_{nt}]\!]$ if there exist n_t and $c_{ncpl.n}$ such that e_l is an element in $[\![concept : c_{ncp}, c_{nf}, concept{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}]\!]$.

An element e_l is an element in $[\![concept : c_{ncp}, s_{tt}, c_{ncpl}]\!]$ if there exist n_t and i_{nt} such that e_l is an element in $[\![concept : c_{ncp}, s_{tt}, concept{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}]\!]$. An element e_l is an element in $[\![concept : c_{ncp}, c_{nf}, c_{ncpl.n}]\!]$ if there exist n_t and i_{nt} such that e_l is an element in $[\![concept : c_{ncp}, c_{nf}, concept{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}]\!]$.

An element e_l is an element in $[\![concept : c_{ncp}, s_{tt}]\!]$ if there exist n_t, i_{nt}, and c_{ncpl} such that e_l is an element in $[\![concept : c_{ncp}, s_{tt}, concept{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}]\!]$. An element e_l is an element in $[\![concept : c_{ncp}, c_{nf}]\!]$ if there exist n_t, i_{nt}, and $c_{ncpl.n}$ such that e_l is an element in $[\![concept : c_{ncp}, c_{nf}, concept{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}]\!]$.

7.2.3. Kinds of orders in the context of concepts

A number n_t is an order in $[\![e_l, concept : c_{ncp}, s_{tt}, concept{-}order :, c_{ncpl}]\!]$ if e_l is an element in $[\![concept : c_{ncp}, s_{tt}, concept{-}order : n_t, c_{ncpl}]\!]$. A number n_t is an order in $[\![e_l, concept : c_{ncp}, c_{nf}, concept{-}order :, c_{ncpl.n}]\!]$ if e_l is an element in $[\![concept : c_{ncp}, c_{nf}, concept{-}order : n_t, c_{ncpl.n}]\!]$.

A number n_t is an order in $[\![e_l, concept : c_{ncp}, s_{tt}, element{-}order : i_{nt}]\!]$ if e_l is an element in $[\![concept : c_{ncp}, s_{tt}, concept{-}order : n_t, element{-}order : i_{nt}]\!]$. A number n_t is an order in $[\![e_l, concept : c_{ncp}, c_{nf}, element{-}order : i_{nt}]\!]$ if e_l is an element in $[\![concept : c_{ncp}, c_{nf}, concept{-}order : n_t, element{-}order : i_{nt}]\!]$.

A number n_t is an order in $[\![e_l, concept : c_{ncp}, s_{tt}, concept{-}order :]\!]$ if e_l is an element in $[\![concept : c_{ncp}, s_{tt}, concept{-}order : n_t]\!]$. A number n_t is an order in $[\![e_l, concept : c_{ncp}, c_{nf}, concept{-}order :]\!]$ if e_l is an element in $[\![concept : c_{ncp}, c_{nf}, concept{-}order : n_t]\!]$.

7.2.4. Kinds of orders in the context of elements

A number i_{nt} is an order in $[\![e_l, concept : c_{ncp}, s_{tt}, element{-}order :, c_{ncpl}]\!]$ if e_l is an element in $[\![concept : c_{ncp}, s_{tt}, element{-}order : i_{nt}, c_{ncpl}]\!]$. A number i_{nt} is an order in $[\![e_l, concept : c_{ncp}, c_{nf}, element{-}order :, c_{ncpl.n}]\!]$ if e_l is an element in $[\![concept : c_{ncp}, c_{nf}, element{-}order : i_{nt}, c_{ncpl.n}]\!]$.

A number i_{nt} is an order in $[\![e_l, concept : c_{ncp}, s_{tt}, concept{-}order : n_t]\!]$ if e_l is an element in $[\![concept : c_{ncp}, s_{tt}, concept{-}order : n_t, element{-}order : i_{nt}]\!]$. A number i_{nt} is an order in

$[\![e_l, concept : c_{ncp}, c_{nf}, concept\text{−}order : n_t]\!]$ if e_l is an element in $[\![concept : c_{ncp}, c_{nf}, concept\text{−}order : n_t, element\text{−}order : i_{nt}]\!]$.

A number i_{nt} is an order in $[\![e_l, concept : c_{ncp}, s_{tt}, element\text{−}order :]\!]$ if e_l is an element in $[\![concept : c_{ncp}, s_{tt}, element\text{−}order : i_{nt}]\!]$. A number i_{nt} is an order in $[\![e_l, concept : c_{ncp}, c_{nf}, element\text{−}order :]\!]$ if e_l is an element in $[\![concept : c_{ncp}, c_{nf}, element\text{−}order : i_{nt}]\!]$.

7.2.5. Kinds of concretizations

A conceptual c_{ncpl} is a concretization in $[\![e_l, concept : c_{ncp}, s_{tt}, concept\text{−}order : n_t]\!]$ if e_l is an element in $[\![concept : c_{ncp}, s_{tt}, concept\text{−}order : n_t, c_{ncpl}]\!]$. A conceptual $c_{ncpl.n}$ is a concretization in $[\![e_l, concept : c_{ncp}, c_{nf}, concept\text{−}order : n_t]\!]$ if e_l is an element in $[\![concept : c_{ncp}, c_{nf}, concept\text{−}order : n_t, c_{ncpl.n}]\!]$.

A conceptual c_{ncpl} is a concretization in $[\![e_l, concept : c_{ncp}, s_{tt}, element\text{−}order : i_{nt}]\!]$ if e_l is an element in $[\![concept : c_{ncp}, s_{tt}, element\text{−}order : i_{nt}, c_{ncpl}]\!]$. A conceptual $c_{ncpl.n}$ is a concretization in $[\![e_l, concept : c_{ncp}, c_{nf}, element\text{−}order : i_{nt}]\!]$ if e_l is an element in $[\![concept : c_{ncp}, c_{nf}, element\text{−}order : i_{nt}, c_{ncpl.n}]\!]$.

A conceptual c_{ncpl} is a concretization in $[\![e_l, concept : c_{ncp}, s_{tt}]\!]$ if e_l is an element in $[\![concept : c_{ncp}, s_{tt}, c_{ncpl}]\!]$. A conceptual $c_{ncpl.n}$ is a concretization in $[\![e_l, concept : c_{ncp}, c_{nf}]\!]$ if e_l is an element in $[\![concept : c_{ncp}, c_{nf}, c_{ncpl.n}]\!]$.

$\bigoplus$ Let $c_{ncl.1} = (-3 : 10, -2 : inch, -1 : area, 0 : e_{l.g.1}, 1 : triangle, 2 : Euclidean, 3 : 2)$, $c_{ncl.2} = (-3 : 2, -2 : cm, -1 : perimeter, 0 : e_{l.g.2}, 1 : rectangle, 2 : Euclidean, 3 : 2)$, and $[support\ s_{tt}] = \{c_{ncl.1}, c_{ncl.2}\}$. Then the following properties hold:

- 10, *inch*, *area*, $e_{l.g.1}$ are elements in $[\![concept : triangle, s_{tt}]\!]$;
- 2, *cm*, *perimeter*, $e_{l.g.2}$ are elements in $[\![concept : rectangle, s_{tt}]\!]$;
- 10, *inch*, *area*, $e_{l.g.1}$, 2, *cm*, *perimeter*, $e_{l.g.2}$, *triangle*, *rectangle* are elements in $[\![concept : Eucludian, s_{tt}]\!]$;
- 10, *inch*, *area*, $e_{l.g.1}$, 2, *cm*, *perimeter*, $e_{l.g.2}$, *triangle*, *rectangle*, *Eucludian* are elements in $[\![concept : 2, s_{tt}]\!]$;
- $c_{ncl.1}$ is a concretization in $[\![concept : triangle]\!]$, $[\![concept : Eucludian]\!]$, $[\![concept : 2]\!]$ in $[\![s_{tt}]\!]$;
- $c_{ncl.2}$ is a concretization in $[\![concept : rectangle]\!]$, $[\![concept : Eucludian]\!]$, $[\![concept : 2]\!]$ in $[\![s_{tt}]\!]$;
- 1 is an order in $[\![e_{l.g.2}, concept : rectangle, s_{tt}, concept\text{−}order :]\!]$;

– 0 is an order in $[\![e_{l.g.1}, concept : triangle, s_{tt}, element\text{–}order :]\!]$;

– -1 is an order in $[\![area, concept : triangle, s_{tt}, element\text{–}order :]\!]$;

– -2 is an order in $[\![cm, concept : Eucludian, s_{tt}, element\text{–}order :]\!]$.

7.3. The property of direct concepts

Proposition 6. If c_{ncp} is a concept in $[\![s_{tt}]\!]$ and e_l is an element in $[\![concept : c_{ncp}, s_{tt}, concept\text{–}order : 1]\!]$, then e_l is either an individual in $[\![s_{tt}]\!]$, or e_l is an attribute in $[\![s_{tt}]\!]$.

Proof. This follows from the definition of direct concepts. □

7.4. The content of concepts

The content of a concept describes its semantics.

A set s_t is the content in $[\![concept : c_{ncp}, s_{tt}, concept\text{–}order : n_t, element\text{–}order : i_{nt}, c_{ncpl}]\!]$ if s_t is the set of all elements in $[\![concept : c_{ncp}, s_{tt}, concept\text{–}order : n_t, element\text{–}order : i_{nt}, c_{ncpl}]\!]$. A set s_t is the content in $[\![concept : c_{ncp}, c_{nf}, concept\text{–}order : n_t, element\text{–}order : i_{nt}, c_{ncpl.n}]\!]$ if s_t is the set of all elements in $[\![concept : c_{ncp}, c_{nf}, concept\text{–}order : n_t, element\text{–}order : i_{nt}, c_{ncpl.n}]\!]$.

A set s_t is the content in $[\![concept : c_{ncp}, s_{tt}, concept\text{–}order : n_t, element\text{–}order : i_{nt}]\!]$ if $s_t = \bigcup_{c_{ncpl}[\![s_{tt}]\!]} s_t[\![concept : c_{ncp}, s_{tt}, concept\text{–}order : n_t, element\text{–}order : i_{nt}, c_{ncpl}]\!]$. A set s_t is the content in $[\![concept : c_{ncp}, c_{nf}, concept\text{–}order : n_t, element\text{–}order : i_{nt}]\!]$ if $s_t = \bigcup_{c_{ncpl.n}[\![c_{nf}]\!]} s_t$ $[\![concept : c_{ncp}, c_{nf}, concept\text{–}order : n_t, element\text{–}order : i_{nt}, c_{ncpl.n}]\!]$.

A set s_t is the content in $[\![concept : c_{ncp}, s_{tt}, concept\text{–}order : n_t]\!]$ if $s_t = \bigcup_{i_{nt}<n_t} s_t[\![concept : c_{ncp}, s_{tt}, concept\text{–}order : n_t, element\text{–}order : i_{nt}]\!]$. A set s_t is the content in $[\![concept : c_{ncp}, c_{nf}, concept\text{–}order : n_t]\!]$ if $s_t = \bigcup_{i_{nt}<n_t} s_t[\![concept : c_{ncp}, c_{nf}, concept\text{–}order : n_t, element\text{–}order : i_{nt}]\!]$.

A set s_t is the content in $[\![concept : c_{ncp}, s_{tt}]\!]$ if $s_t = \bigcup_{n_t} s_t[\![concept : c_{ncp}, s_{tt}, concept\text{–}order : n_t]\!]$. A set s_t is the content in $[\![concept : c_{ncp}, c_{nf}]\!]$ if $s_t = \bigcup_{n_t} s_t[\![concept : c_{ncp}, c_{nf}, concept\text{–}order : n_t]\!]$.

⊕ Let $c_{ncl.1} = (-3 : 10, -2 : inch, -1 : area, 0 : e_{l.g.1}, 1 : triangle, 2 : Euclidean, 3 : 2)$, $c_{ncl.2} = (-3 : 2, -2 : cm, -1 : perimeter, 0 : e_{l.g.2}, 1 : rectangle, 2 : Euclidean, 3 : 2)$, and $[support\ s_{tt}] = \{c_{ncl.1}, c_{ncl.2}\}$. Then the following properties hold:

– $\{10, inch, area, e_{l.g.1}\}$ is the content in $[\![concept : triangle, s_{tt}]\!]$;

– $\{2, cm, perimeter, e_{l.g.2}\}$ is the content in $[\![concept : rectangle, s_{tt}]\!]$;

– $\{10, inch, area, e_{l.g.1}, 2, cm, perimeter, e_{l.g.2}, triangle, rectangle\}$ is the content in $[\![concept : Eucludian, s_{tt}]\!]$;

– $\{10, inch, area, e_{l.g.1}, 2, cm, perimeter, e_{l.g.2}, triangle, rectangle, Eucludian\}$ is the content in $[\![concept : 2, s_{tt}]\!]$.

7.5. Mediators

7.5.1. Mediators, elements, degrees

An element $e_{l.1}$ is a mediator in $[\![e_l, concept : c_{ncp}, s_{tt}, concept{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}]\!]$ if e_l is an element in $[\![concept : c_{ncp}, s_{tt}, concept{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}]\!]$, $e_{l.1}$ is an element in $[\![c_{ncpl}, i_{nt.1}]\!]$, and $i_{nt} < i_{nt.1} < n_t$. It is between e_l and c_{ncp} in c_{ncpl} in the position $i_{nt.1}$, thus separating e_l from c_{ncp} in c_{ncpl}. An element $e_{l.1}$ is a mediator in $[\![e_l, concept : c_{ncp}, c_{nf}, concept{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}]\!]$ if e_l is an element in $[\![concept : c_{ncp}, c_{nf}, concept{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}]\!]$, $e_{l.1}$ is an element in $[\![c_{ncpl.n}, i_{nt.1}]\!]$, and $i_{nt} < i_{nt.1} < n_t$.

An element $e_{l.1}$ is a mediator in $[\![e_l, concept : c_{ncp}, s_{tt}, concept{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}]\!]$ if there exists $i_{nt.1}$ such that $e_{l.1}$ is a mediator in $[\![e_l, concept : c_{ncp}, s_{tt}, concept{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}]\!]$. An element $e_{l.1}$ is a mediator in $[\![e_l, concept : c_{ncp}, c_{nf}, concept{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}]\!]$ if there exists $i_{nt.1}$ such that $e_{l.1}$ is a mediator in $[\![e_l, concept : c_{ncp}, c_{nf}, concept{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}]\!]$.

An element e_l is an element in $[\![concept : c_{ncp}, s_{tt}, concept{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}, mediator{-}degree : n_{at.1}]\!]$ if e_l is an element in $[\![concept : c_{ncp}, s_{tt}, concept{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}]\!]$ and $n_{at.1}$ is the number of orders $i_{nt.1}$ in $[\![c_{ncpl}, \hat{e}_l]\!]$ such that $i_{nt} < i_{nt.1} < n_t$. It is separated from c_{ncp} in c_{ncpl} by $n_{at.1}$ of mediators. An element e_l is an element in $[\![concept : c_{ncp}, c_{nf}, concept{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}, mediator{-}degree : n_{at.1}]\!]$ if e_l is an element in $[\![concept : c_{ncp}, c_{nf}, concept{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}]\!]$ and $n_{at.1}$ is the number of orders $i_{nt.1}$ in $[\![c_{ncpl.n}, \hat{e}_l]\!]$ such that $i_{nt} < i_{nt.1} < n_t$.

A number $n_{at.1}$ is a degree in $[\![e_l, concept : c_{ncp}, s_{tt}, concept{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}, mediator{-}degree :]\!]$ if e_l is an element in $[\![concept : c_{ncp}, s_{tt}, concept{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}, mediator{-}degree : n_{at.1}]\!]$. It specifies how many mediators separate e_l from c_{ncp} in c_{ncpl}. A number $n_{at.1}$ is a degree in $[\![e_l, concept : c_{ncp}, c_{nf}, concept{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}, mediator{-}degree :]\!]$ if e_l is an element in $[\![concept : c_{ncp}, c_{nf}, concept{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}, mediator{-}degree : n_{at.1}]\!]$.

7.5.2. Kinds of elements

An element e_l is an element in $[\![concept : c_{ncp}, s_{tt}, concept{-}order : n_t, element{-}order : i_{nt}, mediator{-}degree : n_{at.1}]\!]$ if there exists c_{ncpl} such that e_l is an element in $[\![concept : c_{ncp}, s_{tt}, concept{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}, mediator{-}degree : n_{at.1}]\!]$. An element e_l is an element in $[\![concept : c_{ncp}, c_{nf}, concept{-}order : n_t, element{-}order : i_{nt}, mediator{-}degree : n_{at.1}]\!]$ if there exists $c_{ncpl.n}$ such that e_l is an element in $[\![concept : c_{ncp}, c_{nf}, concept{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}, mediator{-}degree : n_{at.1}]\!]$.

An element e_l is an element in $[\![concept : c_{ncp}, s_{tt}, concept{-}order : n_t, c_{ncpl}, mediator{-}degree : n_{at.1}]\!]$ if there exists i_{nt} such that e_l is an element in $[\![concept : c_{ncp}, s_{tt}, concept{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}, mediator{-}degree : n_{at.1}]\!]$. An element e_l is an element in $[\![concept : c_{ncp}, c_{nf}, concept{-}order : n_t, c_{ncpl.n}, mediator{-}degree : n_{at.1}]\!]$ if there exists i_{nt} such that e_l is an element in $[\![concept : c_{ncp}, c_{nf}, concept{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}, mediator{-}degree : n_{at.1}]\!]$.

An element e_l is an element in $[\![concept : c_{ncp}, s_{tt}, element{-}order : i_{nt}, c_{ncpl}, mediator{-}degree : n_{at.1}]\!]$ if there exists n_t such that e_l is an element in $[\![concept : c_{ncp}, s_{tt}, concept{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}, mediator{-}degree : n_{at.1}]\!]$. An element e_l is an element in $[\![concept : c_{ncp}, c_{nf}, element{-}order : i_{nt}, c_{ncpl.n}, mediator{-}degree : n_{at.1}]\!]$ if there exists n_t such that e_l is an element in $[\![concept : c_{ncp}, c_{nf}, concept{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}, mediator{-}degree : n_{at.1}]\!]$.

An element e_l is an element in $[\![concept : c_{ncp}, s_{tt}, concept{-}order : n_t, mediator{-}degree : n_{at.1}]\!]$ if there exist i_{nt} and c_{ncpl} such that e_l is an element in $[\![concept : c_{ncp}, s_{tt}, concept{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}, mediator{-}degree : n_{at.1}]\!]$. An element e_l is an element in $[\![concept : c_{ncp}, c_{nf}, concept{-}order : n_t, mediator{-}degree : n_{at.1}]\!]$ if there exist i_{nt} and $c_{ncpl.n}$ such that e_l is an element in $[\![concept : c_{ncp}, c_{nf}, concept{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}, mediator{-}degree : n_{at.1}]\!]$.

An element e_l is an element in $[\![concept : c_{ncp}, s_{tt}, element{-}order : i_{nt}, mediator{-}degree : n_{at.1}]\!]$ if there exist n_t and c_{ncpl} such that e_l is an element in $[\![concept : c_{ncp}, s_{tt}, concept{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}, mediator{-}degree : n_{at.1}]\!]$. An element e_l is an element in $[\![concept : c_{ncp}, c_{nf}, element{-}order : i_{nt}, mediator{-}degree : n_{at.1}]\!]$ if there exist n_t and $c_{ncpl.n}$ such that e_l is an element in $[\![concept : c_{ncp}, c_{nf}, concept{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}, mediator{-}degree : n_{at.1}]\!]$.

An element e_l is an element in $[\![concept : c_{ncp}, s_{tt}, c_{ncpl}, mediator{-}degree : n_{at.1}]\!]$ if there exist

n_t and i_{nt} such that e_l is an element in $[\![concept : c_{ncp}, s_{tt}, concept-order : n_t, element-order : i_{nt}, c_{ncpl}, mediator-degree : n_{at.1}]\!]$. An element e_l is an element in $[\![concept : c_{ncp}, c_{nf}, c_{ncpl.n}, mediator-degree : n_{at.1}]\!]$ if there exist n_t and i_{nt} such that e_l is an element in $[\![concept : c_{ncp}, c_{nf}, concept-order : n_t, element-order : i_{nt}, c_{ncpl.n}, mediator-degree : n_{at.1}]\!]$.

An element e_l is an element in $[\![concept : c_{ncp}, s_{tt}, mediator-degree : n_{at.1}]\!]$ if there exist n_t, i_{nt}, and c_{ncpl} such that e_l is an element in $[\![concept : c_{ncp}, s_{tt}, concept-order : n_t, element-order : i_{nt}, c_{ncpl}, mediator-degree : n_{at.1}]\!]$. An element e_l is an element in $[\![concept : c_{ncp}, c_{nf}, mediator-degree : n_{at.1}]\!]$ if there exist n_t, i_{nt}, and $c_{ncpl.n}$ such that e_l is an element in $[\![concept : c_{ncp}, c_{nf}, concept-order : n_t, element-order : i_{nt}, c_{ncpl.n}, mediator-degree : n_{at.1}]\!]$.

7.5.3. Kinds of degrees

A number $n_{at.1}$ is a degree in $[\![e_l, concept : c_{ncp}, s_{tt}, concept-order : n_t, element-order : i_{nt}, mediator-degree :]\!]$ if e_l is an element in $[\![concept : c_{ncp}, s_{tt}, concept-order : n_t, element-order : i_{nt}, mediator-degree : n_{at.1}]\!]$. A number $n_{at.1}$ is a degree in $[\![e_l, concept : c_{ncp}, c_{nf}, concept-order : n_t, element-order : i_{nt}, mediator-degree :]\!]$ if e_l is an element in $[\![concept : c_{ncp}, c_{nf}, concept-order : n_t, element-order : i_{nt}, mediator-degree : n_{at.1}]\!]$.

A number $n_{at.1}$ is a degree in $[\![e_l, concept : c_{ncp}, s_{tt}, concept-order : n_t, c_{ncpl}, mediator-degree :]\!]$ if e_l is an element in $[\![concept : c_{ncp}, s_{tt}, concept-order : n_t, c_{ncpl}, mediator-degree : n_{at.1}]\!]$. A number $n_{at.1}$ is a degree in $[\![e_l, concept : c_{ncp}, c_{nf}, concept-order : n_t, c_{ncpl.n}, mediator-degree :]\!]$ if e_l is an element in $[\![concept : c_{ncp}, c_{nf}, concept-order : n_t, c_{ncpl.n}, mediator-degree : n_{at.1}]\!]$.

A number $n_{at.1}$ is a degree in $[\![e_l, concept : c_{ncp}, s_{tt}, element-order : i_{nt}, c_{ncpl}, mediator-degree :]\!]$ if e_l is an element in $[\![concept : c_{ncp}, s_{tt}, element-order : i_{nt}, c_{ncpl}, mediator-degree : n_{at.1}]\!]$. A number $n_{at.1}$ is a degree in $[\![e_l, concept : c_{ncp}, c_{nf}, element-order : i_{nt}, c_{ncpl.n}, mediator-degree :]\!]$ if e_l is an element in $[\![concept : c_{ncp}, c_{nf}, element-order : i_{nt}, c_{ncpl.n}, mediator-degree : n_{at.1}]\!]$.

A number $n_{at.1}$ is a degree in $[\![e_l, concept : c_{ncp}, s_{tt}, element-order : i_{nt}, c_{ncpl}, mediator-degree :]\!]$ if e_l is an element in $[\![concept : c_{ncp}, s_{tt}, c_{ncpl}, mediator-degree : n_{at.1}]\!]$. A number $n_{at.1}$ is a degree in $[\![e_l, concept : c_{ncp}, c_{nf}, element-order : i_{nt}, c_{ncpl.n}, mediator-degree :]\!]$ if e_l is an element in $[\![concept : c_{ncp}, c_{nf}, c_{ncpl.n}, mediator-degree : n_{at.1}]\!]$.

A number $n_{at.1}$ is a degree in $[\![e_l, concept : c_{ncp}, s_{tt}, concept-order : n_t, mediator-degree :]\!]$ if e_l is an element in $[\![concept : c_{ncp}, s_{tt}, concept-order : n_t, mediator-degree : n_{at.1}]\!]$. A number $n_{at.1}$ is a degree in $[\![e_l, concept : c_{ncp}, c_{nf}, concept{-}order : n_t, mediator{-}degree :]\!]$ if e_l is an element in $[\![concept : c_{ncp}, c_{nf}, concept{-}order : n_t, mediator{-}degree : n_{at.1}]\!]$.

A number $n_{at.1}$ is a degree in $[\![e_l, concept : c_{ncp}, s_{tt}, element{-}order : i_{nt}, mediator{-}degree :]\!]$ if e_l is an element in $[\![concept : c_{ncp}, s_{tt}, element{-}order : i_{nt}, mediator{-}degree : n_{at.1}]\!]$. A number $n_{at.1}$ is a degree in $[\![e_l, concept : c_{ncp}, c_{nf}, element{-}order : i_{nt}, mediator{-}degree :]\!]$ if e_l is an element in $[\![concept : c_{ncp}, c_{nf}, element{-}order : i_{nt}, mediator{-}degree : n_{at.1}]\!]$.

A number $n_{at.1}$ is a degree in $[\![e_l, concept : c_{ncp}, s_{tt}, mediator{-}degree :]\!]$ if e_l is an element in $[\![concept : c_{ncp}, s_{tt}, mediator{-}degree : n_{at.1}]\!]$. A number $n_{at.1}$ is a degree in $[\![e_l, concept : c_{ncp}, c_{nf}, mediator{-}degree :]\!]$ if e_l is an element in $[\![concept : c_{ncp}, c_{nf}, mediator{-}degree : n_{at.1}]\!]$.

⊕ Let $c_{ncl.1} = (-3 : 10, -2 : inch, -1 : area, 0 : f_g, 1 : triangle, 2 : Euclidean, 3 : 2)$, $c_{ncl.2} = (-3 : 10, -2 : inch, -1 : area, 0 : f_g, 2 : Euclidean, 3 : 2)$, and $[support\ s_{tt}] = \{c_{ncl.1}, c_{ncl.2}\}$. Then f_g is an element in the following contexts:

- $[\![concept : triangle, s_{tt}]\!]$ with the decree 0 and without mediators;
- $[\![concept : Euclidean, s_{tt}]\!]$ with the decree 1 and the mediator *triangle*;
- $[\![concept : 2, s_{tt}]\!]$ with the decree 2 and the mediators *triangle* and *Euclidean*;
- $[\![concept : Euclidean, s_{tt}]\!]$ with the decree 0 and without mediators;
- $[\![concept : 2, s_{tt}]\!]$ with the decree 1 and the mediator *Euclidean*.

7.6. Direct elements

An element e_l is a direct element in $[\![concept : c_{ncp}, s_{tt}, concept{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}]\!]$ if e_l is an element in $[\![concept : c_{ncp}, s_{tt}, concept{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}, mediator{-}degree : 0]\!]$. An element e_l is a direct element in $[\![concept : c_{ncp}, c_{nf}, concept{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}]\!]$ if e_l is an element in $[\![concept : c_{ncp}, c_{nf}, concept{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}, mediator{-}degree : 0]\!]$.

7.6.1. Kinds of direct elements

An element e_l is a direct element in $[\![concept : c_{ncp}, s_{tt}, concept{-}order : n_t, element{-}order : i_{nt}]\!]$ if there exists c_{ncpl} such that e_l is a direct element in $[\![concept : c_{ncp}, s_{tt}, concept{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}]\!]$. An element e_l is a direct element in $[\![concept : c_{ncp}, c_{nf}, concept{-}order : n_t, element{-}order : i_{nt}]\!]$ if there exists c_{ncpl} such that e_l is a direct element in $[\![concept : c_{ncp}, c_{nf}, concept{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}]\!]$.

An element e_l is a direct element in $[\![concept : c_{ncp}, s_{tt}, concept{-}order : n_t, c_{ncpl}]\!]$ if there exists i_{nt} such that e_l is a direct element in $[\![concept : c_{ncp}, s_{tt}, concept{-}order : n_t, element{-}order :$

$i_{nt}, c_{ncpl}]\!]$. An element e_l is a direct element in $[\![concept : c_{ncp}, c_{nf}, concept{-}order : n_t, c_{ncpl.n}]\!]$ if there exists i_{nt} such that e_l is a direct element in $[\![concept : c_{ncp}, c_{nf}, concept{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}]\!]$.

An element e_l is a direct element in $[\![concept : c_{ncp}, s_{tt}, element{-}order : i_{nt}, c_{ncpl}]\!]$ if there exists n_t such that e_l is a direct element in $[\![concept : c_{ncp}, s_{tt}, concept{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}]\!]$. An element e_l is a direct element in $[\![concept : c_{ncp}, c_{nf}, element{-}order : i_{nt}, c_{ncpl.n}]\!]$ if there exists n_t such that e_l is a direct element in $[\![concept : c_{ncp}, c_{nf}, concept{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}]\!]$.

An element e_l is a direct element in $[\![concept : c_{ncp}, s_{tt}, concept{-}order : n_t]\!]$ if there exist i_{nt} and c_{ncpl} such that e_l is a direct element in $[\![concept : c_{ncp}, s_{tt}, concept{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}]\!]$. An element e_l is a direct element in $[\![concept : c_{ncp}, c_{nf}, concept{-}order : n_t]\!]$ if there exist i_{nt} and $c_{ncpl.n}$ such that e_l is a direct element in $[\![concept : c_{ncp}, c_{nf}, concept{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}]\!]$.

An element e_l is a direct element in $[\![concept : c_{ncp}, s_{tt}, element{-}order : i_{nt}]\!]$ if there exist n_t and c_{ncpl} such that e_l is a direct element in $[\![concept : c_{ncp}, s_{tt}, concept{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}]\!]$. An element e_l is a direct element in $[\![concept : c_{ncp}, c_{nf}, element{-}order : i_{nt}]\!]$ if there exist n_t and $c_{ncpl.n}$ such that e_l is a direct element in $[\![concept : c_{ncp}, c_{nf}, concept{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}]\!]$.

An element e_l is a direct element in $[\![concept : c_{ncp}, s_{tt}, c_{ncpl}]\!]$ if there exist n_t and i_{nt} such that e_l is a direct element in $[\![concept : c_{ncp}, s_{tt}, concept{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}]\!]$. An element e_l is a direct element in $[\![concept : c_{ncp}, c_{nf}, c_{ncpl.n}]\!]$ if there exist n_t and i_{nt} such that e_l is a direct element in $[\![concept : c_{ncp}, c_{nf}, concept{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}]\!]$.

An element e_l is a direct element in $[\![concept : c_{ncp}, s_{tt}]\!]$ if there exist n_t, i_{nt}, and c_{ncpl} such that e_l is a direct element in $[\![concept : c_{ncp}, s_{tt}, concept{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}]\!]$. An element e_l is a direct element in $[\![concept : c_{ncp}, c_{nf}]\!]$ if there exist n_t, i_{nt}, and $c_{ncpl.n}$ such that e_l is a direct element in $[\![concept : c_{ncp}, c_{nf}, concept{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}]\!]$.

$\bigoplus$ Let $c_{ncpl} = (-3 : 10, -2 : inch, -1 : area, 0 : f_g, 1 : triangle, 2 : Euclidean, 3 : 2)$ and $s_{tt} = (c_{ncpl} : 3)$. Then the following properties hold:

- f_g is a direct element in $[\![concept : triangle, s_{tt}]\!]$ that means that f_g is a triangle in $[\![s_{tt}]\!]$;
- $triangle$ is a direct element in $[\![concept : Eucludian, s_{tt}]\!]$ that means that classification of geometric figures in Eucludian space includes triangles in $[\![s_{tt}]\!]$;

– *Eucludian* is a direct element in $[\![concept : 2, s_{tt}]\!]$ that means that classification of two-dimensional spaces includes Eucludian space in $[\![s_{tt}]\!]$.

7.7. The direct content of concepts

A set s_t is the direct content in $[\![concept : c_{ncp}, s_{tt}, concept{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}]\!]$ if s_t is the set of all direct elements in $[\![concept : c_{ncp}, s_{tt}, concept{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}]\!]$. A set s_t is the direct content in $[\![concept : c_{ncp}, c_{nf}, concept{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}]\!]$ if s_t is the set of all direct elements in $[\![concept : c_{ncp}, c_{nf}, concept{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}]\!]$.

A set s_t is the direct content in $[\![concept : c_{ncp}, s_{tt}, concept{-}order : n_t, element{-}order : i_{nt}]\!]$ if $s_t = \bigcup_{c_{ncpl}[\![s_{tt}]\!]} s_t[\![concept : c_{ncp}, s_{tt}, concept{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}]\!]$. A set s_t is the direct content in $[\![concept : c_{ncp}, c_{nf}, concept{-}order : n_t, element{-}order : i_{nt}]\!]$ if $s_t = \bigcup_{c_{ncpl.n}[\![c_{nf}]\!]} s_t[\![concept : c_{ncp}, c_{nf}, concept{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}]\!]$.

A set s_t is the direct content in $[\![concept : c_{ncp}, s_{tt}, concept{-}order : n_t]\!]$ if $s_t = \bigcup_{i_{nt}<n_t} s_t[\![concept : c_{ncp}, s_{tt}, concept{-}order : n_t, element{-}order : i_{nt}]\!]$. A set s_t is the direct content in $[\![concept : c_{ncp}, c_{nf}, concept{-}order : n_t]\!]$ if $s_t = \bigcup_{i_{nt}<n_t} s_t[\![concept : c_{ncp}, c_{nf}, concept{-}order : n_t, element{-}order : i_{nt}]\!]$.

A set s_t is the direct content in $[\![concept : c_{ncp}, s_{tt}]\!]$ if $s_t = \bigcup_{n_t} s_t[\![concept : c_{ncp}, s_{tt}, concept{-}order : n_t]\!]$. A set s_t is the direct content in $[\![concept : c_{ncp}, c_{nf}]\!]$ if $s_t = \bigcup_{n_t} s_t[\![concept : c_{ncp}, c_{nf}, concept{-}order : n_t]\!]$.

$\bigoplus$ Let $c_{ncl.1} = (-3 : 10, -2 : inch, -1 : area, 0 : e_{l.g.1}, 1 : triangle, 2 : Euclidean, 3 : 2)$, $c_{ncl.2} = (-3 : 10, -2 : inch, -1 : area, 0 : e_{l.g.2}, 1 : triangle, 2 : Riemannian, 3 : 2)$, $c_{ncl.3} = (-3 : 10, -2 : inch, -1 : area, 0 : e_{l.g.1}, 3 : 2)$, and $[support\ s_{tt}] = \{c_{ncl.1}, c_{ncl.2}, c_{ncl.3}\}$. Then the following properties hold:

– $\{e_{l.g.1}, e_{l.g.2}\}$ is the direct content in $[\![concept : triangle, s_{tt}]\!]$;

– $\{triangle\}$ is the direct content in $[\![concept : Eucludian, s_{tt}]\!]$;

– $\{triangle\}$ is the direct content in $[\![concept : Riemannian, s_{tt}]\!]$;

– $\{Eucludian, Riemannian\}$ is the direct content in $[\![concept : 2, s_{tt}]\!]$;

– $\{e_{l.g.1}\}$ is the direct content in $[\![concept : 2, s_{tt}]\!]$.

7.8. The content of concepts in the context of mediators

A set s_t is the content in $[\![concept : c_{ncp}, s_{tt}, concept{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}, mediator{-}degree : n_{at.1}]\!]$ if s_t is the set of all elements in $[\![concept : c_{ncp}, s_{tt}, concept{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}, mediator{-}degree : n_{at.1}]\!]$. A set s_t is the content in $[\![concept : c_{ncp}, c_{nf}, concept{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}, mediator{-}degree : n_{at.1}]\!]$ if s_t is the set of all elements in $[\![concept : c_{ncp}, c_{nf}, concept{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}, mediator{-}degree : n_{at.1}]\!]$.

A set s_t is the content in $[\![concept : c_{ncp}, s_{tt}, concept{-}order : n_t, element{-}order : i_{nt}, mediator{-}degree : n_{at.1}]\!]$ if $s_t = \bigcup_{c_{ncpl}[\![s_{tt}]\!]} s_t[\![concept : c_{ncp}, s_{tt}, concept{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}, mediator{-}degree : n_{at.1}]\!]$. A set s_t is the content in $[\![concept : c_{ncp}, c_{nf}, concept{-}order : n_t, element{-}order : i_{nt}, mediator{-}degree : n_{at.1}]\!]$ if $s_t = \bigcup_{c_{ncpl.n}[\![c_{nf}]\!]} s_t$ $[\![concept : c_{ncp}, c_{nf}, concept{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}, mediator{-}degree : n_{at.1}]\!]$.

A set s_t is the content in $[\![concept : c_{ncp}, s_{tt}, concept{-}order : n_t, mediator{-}degree : n_{at.1}]\!]$ if $s_t = \bigcup_{i_{nt}<n_t} s_t[\![concept : c_{ncp}, s_{tt}, concept{-}order : n_t, element{-}order : i_{nt}, mediator{-}degree : n_{at.1}]\!]$. A set s_t is the content in $[\![concept : c_{ncp}, c_{nf}, concept{-}order : n_t, mediator{-}degree : n_{at.1}]\!]$ if $s_t = \bigcup_{i_{nt}<n_t} s_t[\![concept : c_{ncp}, c_{nf}, concept{-}order : n_t, element{-}order : i_{nt}, mediator{-}degree : n_{at.1}]\!]$.

A set s_t is the content in $[\![concept : c_{ncp}, s_{tt}, mediator{-}degree : n_{at.1}]\!]$ if $s_t = \bigcup_{n_t} s_t[\![concept : c_{ncp}, s_{tt}, concept{-}order : i_{nt}, mediator{-}degree : n_{at.1}]\!]$. A set s_t is the content in $[\![concept : c_{ncp}, c_{nf}, mediator{-}degree : n_{at.1}]\!]$ if $s_t = \bigcup_{n_t} s_t[\![concept : c_{ncp}, c_{nf}, concept{-}order : i_{nt}, mediator{-}degree : n_{at.1}]\!]$.

$\bigoplus$ Let $c_{ncl.1} = (-3 : 10, -2 : inch, -1 : area, 0 : e_{l.g.1}, 1 : triangle, 2 : Euclidean, 3 : 2)$, $c_{ncl.2} = (-3 : 10, -2 : inch, -1 : area, 0 : e_{l.g.2}, 1 : triangle, 2 : Riemannian, 3 : 2)$, $c_{ncl.3} = (-3 : 10, -2 : inch, -1 : perimeter, 0 : e_{l.g.3}, 2 : Euclidean, 3 : 2)$, and $[support\ s_{tt}] = \{c_{ncl.1}, c_{ncl.2}, c_{ncl.3}\}$. Then the following properties hold:

- $\{e_{l.g.1}, e_{l.g.2}\}$ is the content in $[\![concept : 2, s_{tt}, mediator{-}degree : 2]\!]$;
- $\{e_{l.g.3}\}$ is the content in $[\![concept : 2, s_{tt}, mediator{-}degree : 1]\!]$;
- $\{area\}$ is the content in $[\![concept : 2, s_{tt}, mediator{-}degree : 3]\!]$;
- $\{perimeter\}$ is the content in $[\![concept : 2, s_{tt}, mediator{-}degree : 2]\!]$.

8. Classification and interpretation of concepts

Concepts are classified according to their orders.

8.1. Concepts of the order 1

A concept c_{ncp} in $[\![s_{tt}, 1]\!]$ models a usual concept in $[\![s_{s.q.i}]\!]$. Elements in $[\![concept : c_{ncp}, s_{tt}, concept{-}order : 1$ are attributes and individuals in $[\![s_{tt}]\!]]\!]$.

⊕ Let $c_{ncl.1} = (-3 : 10, -2 : inch, -1 : area, 0 : e_{l.g.1}, 1 : triangle, 2 : Euclidean, 3 : 2)$, $c_{ncl.2} = (-3 : 2, -2 : cm, -1 : perimeter, 0 : e_{l.g.2}, 1 : triangle, 2 : Euclidean, 3 : 2)$, and $[support\ s_{tt}] = \{c_{ncl.1}, c_{ncl.2}\}$. Then the following properties hold:

- the direct concept *triangle* models triangles in $[\![s_{tt}]\!]$;
- the individuals $e_{l.g.1}$ and $e_{l.g.2}$ are elements of the order 0 of the direct concept *triangle* in $[\![s_{tt}]\!]$ that means that $e_{l.g.1}$ and $e_{l.g.2}$ are triangles in $[\![s_{tt}]\!]$;
- the attributes *area* and *perimeter* are elements of the order -1 of the direct concept *triangle* in $[\![s_{tt}]\!]$ that means that classification of numerical characteristics of triangles includes area and perimeter in $[\![s_{tt}]\!]$;
- the attributes *inch* and *cm* are elements of the order -2 of the direct concept *triangle* in $[\![s_{tt}]\!]$ that means that classification of units of measurement of numerical characteristics of triangles includes inches and centimetres in $[\![s_{tt}]\!]$;
- the attributes 10 and 2 are elements of the order -3 of the direct concept *triangle* in $[\![s_{tt}]\!]$ that means that classification of numeral systems for representing values of numerical characteristics of triangles includes decimal and binary systems in $[\![s_{tt}]\!]$.

8.2. Concepts of the order 2

A concept c_{ncp} in $[\![s_{tt}, 2]\!]$ models a concept space in $[\![s_{s.q.i}]\!]$. Elements in $[\![concept : c_{ncp}, s_{tt}, concept{-}order : 2$ are attributes, individuals and direct concepts in $[\![s_{tt}]\!]]\!]$.

⊕ Let $c_{ncl.1} = (-3 : 10, -2 : inch, -1 : area, 0 : e_{l.g.1}, 1 : triangle, 2 : Euclidean, 3 : 2)$, $c_{ncl.2} = (-3 : 2, -2 : cm, -1 : perimeter, 0 : e_{l.g.2}, 1 : square, 2 : Euclidean, 3 : 2)$, and $[support\ s_{tt}] = \{c_{ncl.1}, c_{ncl.2}\}$. Then the following properties hold:

- the concept space *Euclidean* models Euclidean space in $[\![s_{tt}]\!]$;
- the direct concepts *triangle* and *square* are elements of the order 1 of the concept space *Euclidean* in $[\![s_{tt}]\!]$ that means that classification of geometric figures in Euclidean space includes triangles and squares in $[\![s_{tt}]\!]$;
- the individuals $e_{l.g.1}$ and $e_{l.g.2}$ are elements of the order 0 of the concept space *Euclidean* in $[\![s_{tt}]\!]$ that means that $e_{l.g.1}$ and $e_{l.g.2}$ are geometric figures in Euclidean space in $[\![s_{tt}]\!]$;
- the attributes *area* and *perimeter* are elements of the order -1 of the concept space

$Euclidean$ in $[\![s_{tt}]\!]$ that means that classification of numerical characteristics of geometric figures in Euclidean space includes area and perimeter in $[\![s_{tt}]\!]$;

– the attributes $inch$ and cm are elements of the order -2 of the concept space $Euclidean$ in $[\![s_{tt}]\!]$ that means that classification of units of measurement of numerical characteristics of geometric figures in Euclidean space includes inches and centimetres in $[\![s_{tt}]\!]$;

– the attributes 10 and 2 are elements of the order -3 of the concept space $Euclidean$ in $[\![s_{tt}]\!]$ that means that classification of numeral systems for representing values of numerical characteristics of geometric figures in Euclidean space includes decimal and binary systems in $[\![s_{tt}]\!]$.

8.3. Concepts of the order 3

A concept c_{ncp} in $[\![s_{tt}, 3]\!]$ models a space of concept spaces in $[\![s_{s.q.i}]\!]$. Elements in $[\![concept : c_{ncp}, s_{tt}, concept{-}order : 3]\!]$ are attributes, individuals, direct concepts and concept spaces in $[\![s_{tt}]\!]$.

⨁ Let $c_{ncl.1} = (-3 : 10, -2 : inch, -1 : area, 0 : e_{l.g.1}, 1 : triangle, 2 : Euclidean, 3 : 2)$, $c_{ncl.2} = (-3 : 2, -2 : cm, -1 : perimeter, 0 : e_{l.g.2}, 1 : square, 2 : Riemannian, 3 : 2)$, and $[support\ s_{tt}] = \{c_{ncl.1}, c_{ncl.2}\}$. Then the following properties hold:

– the concept space space 2 models two-dimensional space in $[\![s_{tt}]\!]$;

– the concept spaces $Euclidean$ and $Riemannian$ are elements of the order 2 of the concept space space 2 in $[\![s_{tt}]\!]$ that means that classification of two-dimensional spaces includes Euclidean space and Riemannian space in $[\![s_{tt}]\!]$;

– the direct concepts $triangle$ and $square$ are elements of the order 1 of the concept space space 2 in $[\![s_{tt}]\!]$ that means that classification of geometric figures in two-dimensional space includes triangles and squares in $[\![s_{tt}]\!]$;

– the individuals $e_{l.g.1}$ and $e_{l.g.2}$ are elements of the order 0 of the concept space space 2 in $[\![s_{tt}]\!]$ that means that $e_{l.g.1}$ and $e_{l.g.2}$ are geometric figures in two-dimensional space in $[\![s_{tt}]\!]$;

– the attributes $area$ and $perimeter$ are elements of the order -1 of the concept space space 2 in $[\![s_{tt}]\!]$ that means that classification of numerical characteristics of geometric figures in two-dimensional space includes area and perimeter in $[\![s_{tt}]\!]$;

– the attributes $inch$ and cm are elements of the order -2 of the concept space space 2 in $[\![s_{tt}]\!]$ that means that classification of units of measurement of numerical characteristics of geometric figures in two-dimensional space includes inches and centimetres in $[\![s_{tt}]\!]$;

– the attributes 10 and 2 are elements of the order -3 of the concept space space 2 in $[\![s_{tt}]\!]$ that means that classification of numeral systems for representing values of numerical characteristics of geometric figures in two-dimensional space includes decimal and binary systems in $[\![s_{tt}]\!]$.

8.4. Concepts of higher orders

A concept c_{ncp} in $[\![s_{tt}, n_t]\!]$, where $n_t > 3$, is classified and interpreted in the similar way (by the introduction of the space of concept space spaces and so on.).

9. Structure of attributes

Attributes use the same terminology as concepts.

9.1. Direct attributes

The usual attributes in $[\![s_{s.q.i}]\!]$ which are interpreted as characteristics of elements in $[\![s_{s.q.i}]\!]$ are modelled by the special kind of attributes in $[\![s_{tt}]\!]$, direct attributes in $[\![s_{tt}]\!]$.

9.1.1. Direct concepts

An element e_l is a direct attribute in $[\![s_{tt}, c_{ncpl}]\!]$ if e_l is a attribute in $[\![s_{tt}, 1, c_{ncpl}]\!]$. An element e_l is a direct attribute in $[\![c_{nf}, c_{ncpl.n}]\!]$ if e_l is a attribute in $[\![c_{nf}, 1, c_{ncpl.n}]\!]$.

An element e_l is a direct attribute in $[\![s_{tt}]\!]$ if there exists c_{ncpl} such that e_l is a direct attribute in $[\![s_{tt}, c_{ncpl}]\!]$. An element e_l is a direct attribute in $[\![c_{nf}]\!]$ if there exists $c_{ncpl.n}$ such that e_l is a direct attribute in $[\![c_{nf}, c_{ncpl.n}]\!]$.

9.1.2. Concretizations

A conceptual c_{ncpl} is a concretization in $[\![direct\text{–}attribute : e_l, s_{tt}]\!]$ if e_l is a attribute in $[\![s_{tt}, 1, c_{ncpl}]\!]$. A conceptual $c_{ncpl.n}$ is a concretization in $[\![direct\text{–}attribute : e_l, c_{nf}]\!]$ if e_l is a attribute in $[\![c_{nf}, 1, c_{ncpl.n}]\!]$.

A conceptual c_{ncpl} is a concretization in $[\![s_{tt}, direct\text{–}attribute :]\!]$ if there exists e_l such that c_{ncpl} is a concretization in $[\![direct\text{–}attribute : e_l, s_{tt}]\!]$. A conceptual $c_{ncpl.n}$ is a concretization in $[\![c_{nf}, direct\text{–}attribute :]\!]$ if there exists e_l such that $c_{ncpl.n}$ is a concretization in $[\![direct\text{–}$

$attribute : e_l, c_{nf}]\!]$.

⊕ Let $c_{ncl.1} = (-3 : 10, -2 : inch, -1 : area, 0 : f_g, 1 : triangle, 2 : Euclidean, 3 : 2)$, $c_{ncl.2} = (-3 : 10, -2 : inch, -1 : perimeter, 0 : f_g, 1 : rectangle, 2 : Euclidean, 3 : 2)$, and $[support\ s_{tt}] = \{c_{ncl.1}, c_{ncl.2}\}$. Then the following properties hold:

– $area$ and $perimeter$ are direct attributes in s_{tt};

– $c_{ncl.1}$ is a concretization in $[\![direct{-}attribute : area, s_{tt}]\!]$;

– $c_{ncl.2}$ is a concretization in $[\![direct{-}attribute : perimeter, s_{tt}]\!]$.

9.2. Elements of attributes

9.2.1. Elements, orders, concretizations

An element e_l is an element in $[\![attribute : a_{tt}, s_{tt}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}]\!]$ if a_{tt} is an attibute in $[\![s_{tt}, n_t, c_{ncpl}]\!]$, e_l is an element in $[\![c_{ncpl}, i_{nt}]\!]$, and $-n_t < i_{nt}$. An element e_l is an element in $[\![attribute : a_{tt}, c_{nf}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}]\!]$ if a_{tt} is an attibute in $[\![c_{nf}, n_t, c_{ncpl.n}]\!]$, e_l is an element in $[\![c_{ncpl.n}, i_{nt}]\!]$, and $-n_t < i_{nt}$.

Thus, elements of the attribute a_{tt} can be attributes of orders which are less than the order of a_{tt}, individuals and concepts of all orders.

A number n_t is an order in $[\![e_l, attribute : a_{tt}, s_{tt}, element{-}order : i_{nt}, c_{ncpl}]\!]$ if e_l is an element in $[\![attribute : a_{tt}, s_{tt}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}]\!]$. It specifies the order in $[\![c_{ncpl}, a_{tt}]\!]$. A number n_t is an order in $[\![e_l, attribute : a_{tt}, c_{nf}, element{-}order : i_{nt}, c_{ncpl.n}]\!]$ if e_l is an element in $[\![attribute : a_{tt}, c_{nf}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}]\!]$.

A number i_{nt} is an order in $[\![e_l, attribute : a_{tt}, s_{tt}, attribute{-}order : n_t, c_{ncpl}]\!]$ if e_l is an element in $[\![attribute : a_{tt}, s_{tt}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}]\!]$. It specifies the order in $[\![c_{ncpl}, e_l]\!]$. A number i_{nt} is an order in $[\![e_l, attribute : a_{tt}, c_{nf}, attribute{-}order : n_t, c_{ncpl.n}]\!]$ if e_l is an element in $[\![attribute : a_{tt}, c_{nf}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}]\!]$.

A conceptual c_{ncpl} is a concretization in $[\![e_l, attribute : a_{tt}, s_{tt}, attribute{-}order : n_t, element{-}order : i_{nt}]\!]$ if e_l is an element in $[\![attribute : a_{tt}, s_{tt}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}]\!]$. It defines that e_l is an element in $[\![attribute : a_{tt}, s_{tt}, attribute{-}order : n_t, element{-}order : i_{nt}]\!]$. A conceptual $c_{ncpl.n}$ is a concretization in $[\![e_l, attribute : a_{tt}, c_{nf}, attribute{-}order : n_t, element{-}order : i_{nt}]\!]$ if e_l is an element in $[\![attribute : a_{tt}, c_{nf}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}]\!]$.

9.2.2. Kinds of elements

An element e_l is an element in $[\![attribute : a_{tt}, s_{tt}, attribute{-}order : n_t, element{-}order : i_{nt}]\!]$ if there exists c_{ncpl} such that e_l is an element in $[\![attribute : a_{tt}, s_{tt}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}]\!]$. An element e_l is an element in $[\![attribute : a_{tt}, c_{nf}, attribute{-}order : n_t, element{-}order : i_{nt}]\!]$ if there exists $c_{ncpl.n}$ such that e_l is an element in $[\![attribute : a_{tt}, c_{nf}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}]\!]$.

An element e_l is an element in $[\![attribute : a_{tt}, s_{tt}, attribute{-}order : n_t, c_{ncpl}]\!]$ if there exists i_{nt} such that e_l is an element in $[\![attribute : a_{tt}, s_{tt}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}]\!]$. An element e_l is an element in $[\![attribute : a_{tt}, c_{nf}, attribute{-}order : n_t, c_{ncpl.n}]\!]$ if there exists i_{nt} such that e_l is an element in $[\![attribute : a_{tt}, c_{nf}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}]\!]$.

An element e_l is an element in $[\![attribute : a_{tt}, s_{tt}, attribute{-}order : i_{nt}, c_{ncpl}]\!]$ if there exists n_t such that e_l is an element in $[\![attribute : a_{tt}, s_{tt}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}]\!]$. An element e_l is an element in $[\![attribute : a_{tt}, c_{nf}, attribute{-}order : i_{nt}, c_{ncpl.n}]\!]$ if there exists n_t such that e_l is an element in $[\![attribute : a_{tt}, c_{nf}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}]\!]$.

An element e_l is an element in $[\![attribute : a_{tt}, s_{tt}, attribute{-}order : n_t]\!]$ if there exist i_{nt} and c_{ncpl} such that e_l is an element in $[\![attribute : a_{tt}, s_{tt}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}]\!]$. An element e_l is an element in $[\![attribute : a_{tt}, c_{nf}, attribute{-}order : n_t]\!]$ if there exist i_{nt} and $c_{ncpl.n}$ such that e_l is an element in $[\![attribute : a_{tt}, c_{nf}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}]\!]$.

An element e_l is an element in $[\![attribute : a_{tt}, s_{tt}, element{-}order : i_{nt}]\!]$ if there exist n_t and c_{ncpl} such that e_l is an element in $[\![attribute : a_{tt}, s_{tt}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}]\!]$. An element e_l is an element in $[\![attribute : a_{tt}, c_{nf}, element{-}order : i_{nt}]\!]$ if there exist n_t and $c_{ncpl.n}$ such that e_l is an element in $[\![attribute : a_{tt}, c_{nf}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}]\!]$.

An element e_l is an element in $[\![attribute : a_{tt}, s_{tt}, c_{ncpl}]\!]$ if there exist n_t and i_{nt} such that e_l is an element in $[\![attribute : a_{tt}, s_{tt}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}]\!]$. An element e_l is an element in $[\![attribute : a_{tt}, c_{nf}, c_{ncpl.n}]\!]$ if there exist n_t and i_{nt} such that e_l is an element in $[\![attribute : a_{tt}, c_{nf}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}]\!]$.

An element e_l is an element in $[\![attribute : a_{tt}, s_{tt}]\!]$ if there exist n_t, i_{nt}, and c_{ncpl} such that e_l is an element in $[\![attribute : a_{tt}, s_{tt}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}]\!]$. An element e_l is an element in $[\![attribute : a_{tt}, c_{nf}]\!]$ if there exist n_t, i_{nt}, and $c_{ncpl.n}$ such that e_l is an element in $[\![attribute : a_{tt}, c_{nf}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}]\!]$.

9.2.3. Kinds of orders in the context of attributes

A number n_t is an order in $[\![e_l, attribute : a_{tt}, s_{tt}, attribute{-}order :, c_{ncpl}]\!]$ if e_l is an element in $[\![attribute : a_{tt}, s_{tt}, attribute{-}order : n_t, c_{ncpl}]\!]$. A number n_t is an order in $[\![e_l, attribute : a_{tt}, c_{nf}, attribute{-}order :, c_{ncpl.n}]\!]$ if e_l is an element in $[\![attribute : a_{tt}, c_{nf}, attribute{-}order : n_t, c_{ncpl.n}]\!]$.

A number n_t is an order in $[\![e_l, attribute : a_{tt}, s_{tt}, element{-}order : i_{nt}]\!]$ if e_l is an element in $[\![attribute : a_{tt}, s_{tt}, attribute{-}order : n_t, element{-}order : i_{nt}]\!]$. A number n_t is an order in $[\![e_l, attribute : a_{tt}, c_{nf}, element{-}order : i_{nt}]\!]$ if e_l is an element in $[\![attribute : a_{tt}, c_{nf}, attribute{-}order : n_t, element{-}order : i_{nt}]\!]$.

A number n_t is an order in $[\![e_l, attribute : a_{tt}, s_{tt}, attribute{-}order :]\!]$ if e_l is an element in $[\![attribute : a_{tt}, s_{tt}, attribute{-}order : n_t]\!]$. A number n_t is an order in $[\![e_l, attribute : a_{tt}, c_{nf}, attribute{-}order :]\!]$ if e_l is an element in $[\![attribute : a_{tt}, c_{nf}, attribute{-}order : n_t]\!]$.

9.2.4. Kinds of orders in the context of elements

A number i_{nt} is an order in $[\![e_l, attribute : a_{tt}, s_{tt}, element{-}order :, c_{ncpl}]\!]$ if e_l is an element in $[\![attribute : a_{tt}, s_{tt}, element{-}order : i_{nt}, c_{ncpl}]\!]$. A number i_{nt} is an order in $[\![e_l, attribute : a_{tt}, c_{nf}, element{-}order :, c_{ncpl.n}]\!]$ if e_l is an element in $[\![attribute : a_{tt}, c_{nf}, element{-}order : i_{nt}, c_{ncpl.n}]\!]$.

A number i_{nt} is an order in $[\![e_l, attribute : a_{tt}, s_{tt}, attribute{-}order : n_t]\!]$ if e_l is an element in $[\![attribute : a_{tt}, s_{tt}, attribute{-}order : n_t, element{-}order : i_{nt}]\!]$. A number i_{nt} is an order in $[\![e_l, attribute : a_{tt}, c_{nf}, attribute{-}order : n_t]\!]$ if e_l is an element in $[\![attribute : a_{tt}, c_{nf}, attribute{-}order : n_t, element{-}order : i_{nt}]\!]$.

A number i_{nt} is an order in $[\![e_l, attribute : a_{tt}, s_{tt}, element{-}order :]\!]$ if e_l is an element in $[\![attribute : a_{tt}, s_{tt}, element{-}order : i_{nt}]\!]$. A number i_{nt} is an order in $[\![e_l, attribute : a_{tt}, c_{nf}, element{-}order :]\!]$ if e_l is an element in $[\![attribute : a_{tt}, c_{nf}, element{-}order : i_{nt}]\!]$.

9.2.5. Kinds of concretizations

A conceptual c_{ncpl} is a concretization in $[\![e_l, attribute : a_{tt}, s_{tt}, attribute{-}order : n_t]\!]$ if e_l is an element in $[\![attribute : a_{tt}, s_{tt}, attribute{-}order : n_t, c_{ncpl}]\!]$. A conceptual $c_{ncpl.n}$ is a concretization in $[\![e_l, attribute : a_{tt}, c_{nf}, attribute{-}order : n_t]\!]$ if e_l is an element in $[\![attribute : a_{tt}, c_{nf}, attribute{-}order : n_t, c_{ncpl.n}]\!]$.

A conceptual c_{ncpl} is a concretization in $[\![e_l, attribute : a_{tt}, s_{tt}, element\!-\!order : i_{nt}]\!]$ if e_l is an element in $[\![attribute : a_{tt}, s_{tt}, element\!-\!order : i_{nt}, c_{ncpl}]\!]$. A conceptual $c_{ncpl.n}$ is a concretization in $[\![e_l, attribute : a_{tt}, c_{nf}, element\!-\!order : i_{nt}]\!]$ if e_l is an element in $[\![attribute : a_{tt}, c_{nf}, element\!-\!order : i_{nt}, c_{ncpl.n}]\!]$.

A conceptual c_{ncpl} is a concretization in $[\![e_l, attribute : a_{tt}, s_{tt}]\!]$ if e_l is an element in $[\![attribute : a_{tt}, s_{tt}, c_{ncpl}]\!]$. A conceptual $c_{ncpl.n}$ is a concretization in $[\![e_l, attribute : a_{tt}, c_{nf}]\!]$ if e_l is an element in $[\![attribute : a_{tt}, c_{nf}, c_{ncpl.n}]\!]$.

$\bigoplus$ Let $c_{ncl.1} = (-3 : 10, -2 : inch, -1 : area, 0 : e_{l.g.1}, 1 : triangle, 2 : Euclidean, 3 : 2)$, $c_{ncl.2} = (-3 : 10, -2 : inch, -1 : volume, 0 : e_{l.g.2}, 1 : pyramid, 2 : Riemannian, 3 : 3)$, and $[support\ s_{tt}] = \{c_{ncl.1}, c_{ncl.2}\}$. Then the following properties hold:

- 2, *Euclidean*, *triangle*, $e_{l.g.1}$ are elements in $[\![attribute : area, s_{tt}]\!]$;
- 3, *Riemannian*, *pyramid*, $e_{l.g.2}$ are elements in $[\![attribute : volume, s_{tt}]\!]$;
- 2, *Euclidean*, *triangle*, $e_{l.g.1}$, 3, *Riemannian*, *pyramid*, $e_{l.g.2}$, *area*, *volume* are elements in $[\![attribute : inch, s_{tt}]\!]$;
- 2, *Euclidean*, *triangle*, $e_{l.g.1}$, 3, *Riemannian*, *pyramid*, $e_{l.g.2}$, *area*, *volume*, *inch* are elements in $[\![attribute : 10, s_{tt}]\!]$;
- $c_{ncl.1}$ is a concretization in $[\![attribute : area]\!]$, $[\![attribute : inch]\!]$, $[\![attribute : 10]\!]$ in $[\![s_{tt}]\!]$;
- $c_{ncl.2}$ is a concretization in $[\![attribute : volume]\!]$, $[\![attribute : inch]\!]$, $[\![attribute : 10]\!]$ in $[\![s_{tt}]\!]$;
- 1 is an order in $[\![e_{l.g.2}, attribute : volume, s_{tt}, attribute\!-\!order :]\!]$;
- 0 is an order in $[\![e_{l.g.1}, attribute : area, s_{tt}, element\!-\!order :]\!]$;
- 1 is an order in $[\![triangle, attribute : area, s_{tt}, element\!-\!order :]\!]$;
- 2 is an order in $[\![Eucludian, attribute : inch, s_{tt}, element\!-\!order :]\!]$.

9.3. The property of direct attributes

Proposition 7. If a_{tt} is an attribute in $[\![s_{tt}]\!]$ and e_l is an element in $[\![attribute : a_{tt}, s_{tt}, attribute\!-\!order : 1]\!]$, then e_l is either an individual or e_l is a concept in $[\![s_{tt}]\!]$.

Proof. This follows from the definition of direct attributes. □

9.4. The content of attributes

The content of a attributes describes its semantics.

A set s_t is the content in $[\![attribute : a_{tt}, s_{tt}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}]\!]$ if s_t is the set of all elements in $[\![attribute : a_{tt}, s_{tt}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}]\!]$. A set s_t is the content in $[\![attribute : a_{tt}, c_{nf}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}]\!]$ if s_t is the set of all elements in $[\![attribute : a_{tt}, c_{nf}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}]\!]$.

A set s_t is the content in $[\![attribute : a_{tt}, s_{tt}, attribute{-}order : n_t, element{-}order : i_{nt}]\!]$ if $s_t = \bigcup_{c_{ncpl}[\![s_{tt}]\!]} s_t[\![attribute : a_{tt}, s_{tt}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}]\!]$. A set s_t is the content in $[\![attribute : a_{tt}, c_{nf}, attribute{-}order : n_t, element{-}order : i_{nt}]\!]$ if $s_t = \bigcup_{c_{ncpl.n}[\![c_{nf}]\!]} s_t$ $[\![attribute : a_{tt}, c_{nf}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}]\!]$.

A set s_t is the content in $[\![attribute : a_{tt}, s_{tt}, attribute{-}order : n_t]\!]$ if $s_t = \bigcup_{-n_t<i_{nt}} s_t[\![attribute : a_{tt}, s_{tt}, attribute{-}order : n_t, element{-}order : i_{nt}]\!]$. A set s_t is the content in $[\![attribute : a_{tt}, c_{nf}, attribute{-}order : n_t]\!]$ if $s_t = \bigcup_{-n_t<i_{nt}} s_t[\![attribute : a_{tt}, c_{nf}, attribute{-}order : n_t, element{-}order : i_{nt}]\!]$.

A set s_t is the content in $[\![attribute : a_{tt}, s_{tt}]\!]$ if $s_t = \bigcup_{n_t} s_t[\![attribute : a_{tt}, s_{tt}, attribute{-}order : n_t]\!]$. A set s_t is the content in $[\![attribute : a_{tt}, c_{nf}]\!]$ if $s_t = \bigcup_{n_t} s_t[\![attribute : a_{tt}, c_{nf}, attribute{-}order : n_t]\!]$.

⊕ Let $c_{ncl.1} = (-3 : 10, -2 : inch, -1 : area, 0 : e_{l.g.1}, 1 : triangle, 2 : Euclidean, 3 : 2)$, $c_{ncl.2} = (-3 : 10, -2 : inch, -1 : volume, 0 : e_{l.g.2}, 1 : pyramid, 2 : Riemannian, 3 : 3)$, and $[support\ s_{tt}] = \{c_{ncl.1}, c_{ncl.2}\}$. Then the following properties hold:

- $\{2, Euclidean, triangle, e_{l.g.1}\}$ is the content in $[\![attribute : area, s_{tt}]\!]$;
- $\{3, Riemannian, pyramid, e_{l.g.2}\}$ is the content in $[\![attribute : volume, s_{tt}]\!]$;
- $\{2, Euclidean, triangle, e_{l.g.1}, 3, Riemannian, pyramid, e_{l.g.2}, area, volume\}$ is the content in $[\![attribute : inch, s_{tt}]\!]$;
- $\{2, Euclidean, triangle, e_{l.g.1}, 3, Riemannian, pyramid, e_{l.g.2}, area, volume, inch\}$ is the content in $[\![concept : 10, s_{tt}]\!]$.

9.5. Mediators

9.5.1. Mediators, elements, degrees

An element $e_{l.1}$ is a mediator in $[\![e_l, attribute : a_{tt}, s_{tt}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}, i_{nt.1}]\!]$ if e_l is an element in $[\![attribute : a_{tt}, s_{tt}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}]\!]$, $e_{l.1}$ is an element in $[\![c_{ncpl}, i_{nt.1}]\!]$, and $-n_t < i_{nt.1} < i_{nt}$. It is between a_{tt} and e_l in c_{ncpl} in the position $i_{nt.1}$, thus separating e_l from a_{tt} in c_{ncpl}. An element $e_{l.1}$ is a mediator in

$[\![e_l, attribute : a_{tt}, c_{nf}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}, i_{nt.1}]\!]$ if e_l is an element in $[\![attribute : a_{tt}, c_{nf}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}]\!]$, $e_{l.1}$ is an element in $[\![c_{ncpl.n}, i_{nt.1}]\!]$, and $-n_t < i_{nt.1} < i_{nt}$.

An element $e_{l.1}$ is a mediator in $[\![e_l, attribute : a_{tt}, s_{tt}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}]\!]$ if there exists $i_{nt.1}$ such that $e_{l.1}$ is a mediator in $[\![e_l, attribute : a_{tt}, s_{tt}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}, i_{nt.1}]\!]$. An element $e_{l.1}$ is a mediator in $[\![e_l, attribute : a_{tt}, c_{nf}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}]\!]$ if there exists $i_{nt.1}$ such that $e_{l.1}$ is a mediator in $[\![e_l, attribute : a_{tt}, c_{nf}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}, i_{nt.1}]\!]$.

An element e_l is an element in $[\![attribute : a_{tt}, s_{tt}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}, mediator{-}degree : n_{at.1}]\!]$ if e_l is an element in $[\![attribute : a_{tt}, s_{tt}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}]\!]$ and $n_{at.1}$ is the number of orders $i_{nt.1}$ in $[\![c_{ncpl}, \hat{e}_l]\!]$ such that $-n_t < i_{nt.1} < i_{nt}$. It is separated from a_{tt} in c_{ncpl} by $n_{at.1}$ of mediators. An element e_l is an element in $[\![attribute : a_{tt}, c_{nf}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}, mediator{-}degree : n_{at.1}]\!]$ if e_l is an element in $[\![attribute : a_{tt}, c_{nf}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}]\!]$ and $n_{at.1}$ is the number of orders $i_{nt.1}$ in $[\![c_{ncpl.n}, \hat{e}_l]\!]$ such that $-n_t < i_{nt.1} < i_{nt}$.

A number $n_{at.1}$ is a degree in $[\![e_l, attribute : a_{tt}, s_{tt}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}, mediator{-}degree :]\!]$ if e_l is an element in $[\![attribute : a_{tt}, s_{tt}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}, mediator{-}degree : n_{at.1}]\!]$. It specifies how many mediators separate e_l from a_{tt} in c_{ncpl}. A number $n_{at.1}$ is a degree in $[\![e_l, attribute : a_{tt}, c_{nf}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}, mediator{-}degree :]\!]$ if e_l is an element in $[\![attribute : a_{tt}, c_{nf}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}, mediator{-}degree : n_{at.1}]\!]$.

9.5.2. Kinds of elements

An element e_l is an element in $[\![attribute : a_{tt}, s_{tt}, attribute{-}order : n_t, element{-}order : i_{nt}, mediator{-}degree : n_{at.1}]\!]$ if there exists c_{ncpl} such that e_l is an element in $[\![attribute : a_{tt}, s_{tt}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}, mediator{-}degree : n_{at.1}]\!]$. An element e_l is an element in $[\![attribute : a_{tt}, c_{nf}, attribute{-}order : n_t, element{-}order : i_{nt}, mediator{-}degree : n_{at.1}]\!]$ if there exists $c_{ncpl.n}$ such that e_l is an element in $[\![attribute : a_{tt}, c_{nf}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}, mediator{-}degree : n_{at.1}]\!]$.

An element e_l is an element in $[\![attribute : a_{tt}, s_{tt}, attribute{-}order : n_t, c_{ncpl}, mediator{-}degree : n_{at.1}]\!]$ if there exists i_{nt} such that e_l is an element in $[\![attribute : a_{tt}, s_{tt}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}, mediator{-}degree : n_{at.1}]\!]$. An element e_l is an element in $[\![attribute : a_{tt}, c_{nf}, attribute{-}order : n_t, c_{ncpl.n}, mediator{-}degree : n_{at.1}]\!]$ if there exists i_{nt} such that e_l is an element in $[\![attribute : a_{tt}, c_{nf}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}, mediator{-}degree : n_{at.1}]\!]$.

An element e_l is an element in $[\![attribute : a_{tt}, s_{tt}, element{-}order : i_{nt}, c_{ncpl}, mediator{-}degree : n_{at.1}]\!]$ if there exists n_t such that e_l is an element in $[\![attribute : a_{tt}, s_{tt}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}, mediator{-}degree : n_{at.1}]\!]$. An element e_l is an element in $[\![attribute : a_{tt}, c_{nf}, element{-}order : i_{nt}, c_{ncpl.n}, mediator{-}degree : n_{at.1}]\!]$ if there exists n_t such that e_l is an element in $[\![attribute : a_{tt}, c_{nf}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}, mediator{-}degree : n_{at.1}]\!]$.

An element e_l is an element in $[\![attribute : a_{tt}, s_{tt}, attribute{-}order : n_t, mediator{-}degree : n_{at.1}]\!]$ if there exist i_{nt} and c_{ncpl} such that e_l is an element in $[\![attribute : a_{tt}, s_{tt}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}, mediator{-}degree : n_{at.1}]\!]$. An element e_l is an element in $[\![attribute : a_{tt}, c_{nf}, attribute{-}order : n_t, mediator{-}degree : n_{at.1}]\!]$ if there exist i_{nt} and $c_{ncpl.n}$ such that e_l is an element in $[\![attribute : a_{tt}, c_{nf}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}, mediator{-}degree : n_{at.1}]\!]$.

An element e_l is an element in $[\![attribute : a_{tt}, s_{tt}, element{-}order : i_{nt}, mediator{-}degree : n_{at.1}]\!]$ if there exist n_t and c_{ncpl} such that e_l is an element in $[\![attribute : a_{tt}, s_{tt}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}, mediator{-}degree : n_{at.1}]\!]$. An element e_l is an element in $[\![attribute : a_{tt}, c_{nf}, element{-}order : i_{nt}, mediator{-}degree : n_{at.1}]\!]$ if there exist n_t and $c_{ncpl.n}$ such that e_l is an element in $[\![attribute : a_{tt}, c_{nf}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}, mediator{-}degree : n_{at.1}]\!]$.

An element e_l is an element in $[\![attribute : a_{tt}, s_{tt}, c_{ncpl}, mediator{-}degree : n_{at.1}]\!]$ if there exist n_t and i_{nt} such that e_l is an element in $[\![attribute : a_{tt}, s_{tt}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}, mediator{-}degree : n_{at.1}]\!]$. An element e_l is an element in $[\![attribute : a_{tt}, c_{nf}, c_{ncpl.n}, mediator{-}degree : n_{at.1}]\!]$ if there exist n_t and i_{nt} such that e_l is an element in $[\![attribute : a_{tt}, c_{nf}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}, mediator{-}degree : n_{at.1}]\!]$.

An element e_l is an element in $[\![attribute : a_{tt}, s_{tt}, mediator{-}degree : n_{at.1}]\!]$ if there exist n_t, i_{nt}, and c_{ncpl} such that e_l is an element in $[\![attribute : a_{tt}, s_{tt}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}, mediator{-}degree : n_{at.1}]\!]$. An element e_l is an element in $[\![attribute : a_{tt}, c_{nf}, mediator{-}degree : n_{at.1}]\!]$ if there exist n_t, i_{nt}, and $c_{ncpl.n}$ such that e_l is an element in $[\![attribute : a_{tt}, c_{nf}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}, mediator{-}degree : n_{at.1}]\!]$.

9.5.3. Kinds of degrees

A number $n_{at.1}$ is a degree in $[\![e_l, attribute : a_{tt}, s_{tt}, attribute{-}order : n_t, element{-}order : i_{nt}, mediator{-}degree :]\!]$ if e_l is an element in $[\![attribute : a_{tt}, s_{tt}, attribute{-}order : n_t, element{-}order : i_{nt}, mediator{-}degree : n_{at.1}]\!]$. A number $n_{at.1}$ is a degree in $[\![e_l, attribute : a_{tt}, c_{nf}, attribute{-}order : n_t, element{-}order : i_{nt}, mediator{-}degree :]\!]$ if e_l is an element in $[\![attribute : a_{tt}, c_{nf}, attribute{-}order : n_t, element{-}order : i_{nt}, mediator{-}degree : n_{at.1}]\!]$.

A number $n_{at.1}$ is a degree in $[\![e_l, attribute : a_{tt}, s_{tt}, attribute{-}order : n_t, c_{ncpl}, mediator{-}degree :]\!]$ if e_l is an element in $[\![attribute : a_{tt}, s_{tt}, attribute{-}order : n_t, c_{ncpl}, mediator{-}degree : n_{at.1}]\!]$. A number $n_{at.1}$ is a degree in $[\![e_l, attribute : a_{tt}, c_{nf}, attribute{-}order : n_t, c_{ncpl.n}, mediator{-}degree :]\!]$ if e_l is an element in $[\![attribute : a_{tt}, c_{nf}, attribute{-}order : n_t, c_{ncpl.n}, mediator{-}degree : n_{at.1}]\!]$.

A number $n_{at.1}$ is a degree in $[\![e_l, attribute : a_{tt}, s_{tt}, element{-}order : i_{nt}, c_{ncpl}, mediator{-}degree :]\!]$ if e_l is an element in $[\![attribute : a_{tt}, s_{tt}, element{-}order : i_{nt}, c_{ncpl}, mediator{-}degree : n_{at.1}]\!]$. A number $n_{at.1}$ is a degree in $[\![e_l, attribute : a_{tt}, c_{nf}, element{-}order : i_{nt}, c_{ncpl.n}, mediator{-}degree :]\!]$ if e_l is an element in $[\![attribute : a_{tt}, c_{nf}, element{-}order : i_{nt}, c_{ncpl.n}, mediator{-}degree : n_{at.1}]\!]$.

A number $n_{at.1}$ is a degree in $[\![e_l, attribute : a_{tt}, s_{tt}, element{-}order : i_{nt}, c_{ncpl}, mediator{-}degree :]\!]$ if e_l is an element in $[\![attribute : a_{tt}, s_{tt}, c_{ncpl}, mediator{-}degree : n_{at.1}]\!]$. A number $n_{at.1}$ is a degree in $[\![e_l, attribute : a_{tt}, c_{nf}, element{-}order : i_{nt}, c_{ncpl.n}, mediator{-}degree :]\!]$ if e_l is an element in $[\![attribute : a_{tt}, c_{nf}, c_{ncpl.n}, mediator{-}degree : n_{at.1}]\!]$.

A number $n_{at.1}$ is a degree in $[\![e_l, attribute : a_{tt}, s_{tt}, attribute{-}order : n_t, mediator{-}degree :]\!]$ if e_l is an element in $[\![attribute : a_{tt}, s_{tt}, attribute{-}order : n_t, mediator{-}degree : n_{at.1}]\!]$. A number $n_{at.1}$ is a degree in $[\![e_l, attribute : a_{tt}, c_{nf}, attribute{-}order : n_t, mediator{-}degree :]\!]$ if e_l is an element in $[\![attribute : a_{tt}, c_{nf}, attribute{-}order : n_t, mediator{-}degree : n_{at.1}]\!]$.

A number $n_{at.1}$ is a degree in $[\![e_l, attribute : a_{tt}, s_{tt}, element{-}order : i_{nt}, mediator{-}degree :]\!]$ if e_l is an element in $[\![attribute : a_{tt}, s_{tt}, element{-}order : i_{nt}, mediator{-}degree : n_{at.1}]\!]$. A number $n_{at.1}$ is a degree in $[\![e_l, attribute : a_{tt}, c_{nf}, element{-}order : i_{nt}, mediator{-}degree :]\!]$ if e_l is an element in $[\![attribute : a_{tt}, c_{nf}, element{-}order : i_{nt}, mediator{-}degree : n_{at.1}]\!]$.

A number $n_{at.1}$ is a degree in $[\![e_l, attribute : a_{tt}, s_{tt}, mediator{-}degree :]\!]$ if e_l is an element in $[\![attribute : a_{tt}, s_{tt}, mediator{-}degree : n_{at.1}]\!]$. A number $n_{at.1}$ is a degree in $[\![e_l, attribute : a_{tt}, c_{nf}, mediator{-}degree :]\!]$ if e_l is an element in $[\![attribute : a_{tt}, c_{nf}, mediator{-}degree : n_{at.1}]\!]$.

$\bigoplus$ Let $c_{ncl.1} = (-3 : 10, -2 : inch, -1 : area, 0 : f_g, 1 : triangle, 2 : Euclidean, 3 : 2)$, $c_{ncl.2} = (-3 : 10, -2 : cm, 0 : f_g, 1 : triangle, 2 : Euclidean, 3 : 2)$, and $[support\ s_{tt}] =$

$\{c_{ncl.1}, c_{ncl.2}\}$. Then f_g is an element in the following contexts:

– $[\![attribute : area, s_{tt}]\!]$ with the decree 0 and without mediators;

– $[\![attribute : inch, s_{tt}]\!]$ with the decree 1 and the mediator *area*;

– $[\![attribute : 10, s_{tt}]\!]$ with the decree 2 and the mediators *area* and *inch*;

– $[\![attribute : cm, s_{tt}]\!]$ with the decree 0 and without mediators;

– $[\![attribute : 10, s_{tt}]\!]$ with the decree 1 and the mediator *cm*.

9.6. Direct elements

An element e_l is a direct element in $[\![attribute : a_{tt}, s_{tt}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}]\!]$ if e_l is an element in $[\![attribute : a_{tt}, s_{tt}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}, mediator{-}degree : 0]\!]$. An element e_l is a direct element in $[\![attribute : a_{tt}, c_{nf}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}]\!]$ if e_l is an element in $[\![attribute : a_{tt}, c_{nf}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}, mediator{-}degree : 0]\!]$.

9.6.1. Kinds of direct elements

An element e_l is a direct element in $[\![attribute : a_{tt}, s_{tt}, attribute{-}order : n_t, element{-}order : i_{nt}]\!]$ if there exists c_{ncpl} such that e_l is a direct element in $[\![attribute : a_{tt}, s_{tt}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}]\!]$. An element e_l is a direct element in $[\![attribute : a_{tt}, c_{nf}, attribute{-}order : n_t, element{-}order : i_{nt}]\!]$ if there exists $c_{ncpl.n}$ such that e_l is a direct element in $[\![attribute : a_{tt}, c_{nf}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}]\!]$.

An element e_l is a direct element in $[\![attribute : a_{tt}, s_{tt}, attribute{-}order : n_t, c_{ncpl}]\!]$ if there exists i_{nt} such that e_l is a direct element in $[\![attribute : a_{tt}, s_{tt}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}]\!]$. An element e_l is a direct element in $[\![attribute : a_{tt}, c_{nf}, attribute{-}order : n_t, c_{ncpl.n}]\!]$ if there exists i_{nt} such that e_l is a direct element in $[\![attribute : a_{tt}, c_{nf}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}]\!]$.

An element e_l is a direct element in $[\![attribute : a_{tt}, s_{tt}, element{-}order : i_{nt}, c_{ncpl}]\!]$ if there exists n_t such that e_l is a direct element in $[\![attribute : a_{tt}, s_{tt}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}]\!]$. An element e_l is a direct element in $[\![attribute : a_{tt}, c_{nf}, element{-}order : i_{nt}, c_{ncpl.n}]\!]$ if there exists n_t such that e_l is a direct element in $[\![attribute : a_{tt}, c_{nf}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}]\!]$.

An element e_l is a direct element in $[\![attribute : a_{tt}, s_{tt}, attribute{-}order : n_t]\!]$ if there exist i_{nt} and c_{ncpl} such that e_l is a direct element in $[\![attribute : a_{tt}, s_{tt}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}]\!]$. An element e_l is a direct element in $[\![attribute : a_{tt}, c_{nf},$

$attribute{-}order : n_t]\!]$ if there exist i_{nt} and $c_{ncpl.n}$ such that e_l is a direct element in $[\![attribute : a_{tt}, c_{nf}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}]\!]$.

An element e_l is a direct element in $[\![attribute : a_{tt}, s_{tt}, element{-}order : i_{nt}]\!]$ if there exist n_t and c_{ncpl} such that e_l is a direct element in $[\![attribute : a_{tt}, s_{tt}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}]\!]$. An element e_l is a direct element in $[\![attribute : a_{tt}, c_{nf}, element{-}order : i_{nt}]\!]$ if there exist n_t and $c_{ncpl.n}$ such that e_l is a direct element in $[\![attribute : a_{tt}, c_{nf}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}]\!]$.

An element e_l is a direct element in $[\![attribute : a_{tt}, s_{tt}, c_{ncpl}]\!]$ if there exist n_t and i_{nt} such that e_l is a direct element in $[\![attribute : a_{tt}, s_{tt}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}]\!]$. An element e_l is a direct element in $[\![attribute : a_{tt}, c_{nf}, c_{ncpl.n}]\!]$ if there exist n_t and i_{nt} such that e_l is a direct element in $[\![attribute : a_{tt}, c_{nf}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}]\!]$.

An element e_l is a direct element in $[\![attribute : a_{tt}, s_{tt}]\!]$ if there exist n_t, i_{nt}, and c_{ncpl} such that e_l is a direct element in $[\![attribute : a_{tt}, s_{tt}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}]\!]$. An element e_l is a direct element in $[\![attribute : a_{tt}, c_{nf}]\!]$ if there exist n_t, i_{nt}, and $c_{ncpl.n}$ such that e_l is a direct element in $[\![attribute : a_{tt}, c_{nf}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}]\!]$.

⊕ Let $c_{ncpl} = (-3 : 10, -2 : inch, -1 : area, 0 : f_g, 1 : triangle, 2 : Euclidean, 3 : 2)$ and $s_{tt} = (c_{ncpl} : 3)$. Then the following properties hold:

- f_g is a direct element in $[\![attribute : area, s_{tt}]\!]$ that means that classification of numerical characteristics of f_g includes area in $[\![s_{tt}]\!]$;
- $area$ is a direct element in $[\![attribute : inch, s_{tt}]\!]$ that means that classification of units of measurement of numerical characteristics of geometric figures includes inches in $[\![s_{tt}]\!]$;
- $inch$ is a direct element in $[\![attribute : 10, s_{tt}]\!]$ that means that classification of numeral systems for representing values of numerical characteristics of geometric figures includes decimal system in $[\![s_{tt}]\!]$.

9.7. The direct content of attributes

A set s_t is the direct content in $[\![attribute : a_{tt}, s_{tt}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}]\!]$ if s_t is the set of all direct elements in $[\![attribute : a_{tt}, s_{tt}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}]\!]$. A set s_t is the direct content in $[\![attribute : a_{tt}, c_{nf}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}]\!]$ if s_t is the set of all direct elements in $[\![attribute : a_{tt}, c_{nf}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}]\!]$.

A set s_t is the direct content in $[\![attribute : a_{tt}, s_{tt}, attribute{-}order : n_t, element{-}order : i_{nt}]\!]$ if $s_t = \bigcup_{c_{ncpl}[\![s_{tt}]\!]} s_t[\![attribute : a_{tt}, s_{tt}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}]\!]$. A set s_t is the direct content in $[\![attribute : a_{tt}, c_{nf}, attribute{-}order : n_t, element{-}order : i_{nt}]\!]$ if $s_t = \bigcup_{c_{ncpl.n}[\![c_{nf}]\!]} s_t[\![attribute : a_{tt}, c_{nf}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}]\!]$.

A set s_t is the direct content in $[\![attribute : a_{tt}, s_{tt}, attribute{-}order : n_t]\!]$ if $s_t = \bigcup_{-n_t<i_{nt}} s_t[\![attribute : a_{tt}, s_{tt}, attribute{-}order : n_t, element{-}order : i_{nt}]\!]$. A set s_t is the direct content in $[\![attribute : a_{tt}, c_{nf}, attribute{-}order : n_t]\!]$ if $s_t = \bigcup_{-n_t<i_{nt}} s_t[\![attribute : a_{tt}, c_{nf}, attribute{-}order : n_t, element{-}order : i_{nt}]\!]$.

A set s_t is the direct content in $[\![attribute : a_{tt}, s_{tt}]\!]$ if $s_t = \bigcup_{n_t} s_t[\![attribute : a_{tt}, s_{tt}, attribute{-}order : n_t]\!]$. A set s_t is the direct content in $[\![attribute : a_{tt}, c_{nf}]\!]$ if $s_t = \bigcup_{n_t} s_t[\![attribute : a_{tt}, c_{nf}, attribute{-}order : n_t]\!]$.

$\bigoplus$ Let $c_{ncl.1} = (-3 : 10, -2 : inch, -1 : area, 0 : e_{l.g.1}, 1 : triangle, 2 : Euclidean, 3 : 2)$, $c_{ncl.2} = (-3 : 10, -2 : cm, -1 : area, 0 : e_{l.g.2}, 1 : triangle, 2 : Euclidean, 3 : 2)$, $c_{ncl.3} = (-3 : 10, 0 : e_{l.g.1}, 1 : triangle, 2 : Euclidean, 3 : 2)$, and $[support\ s_{tt}] = \{c_{ncl.1}, c_{ncl.2}, c_{ncl.3}\}$. Then the following properties hold:

- $\{e_{l.g.1}, e_{l.g.2}\}$ is the direct content in $[\![attribute : area, s_{tt}]\!]$;
- $\{area\}$ is the direct content in $[\![attribute : inch, s_{tt}]\!]$;
- $\{area\}$ is the direct content in $[\![attribute : cm, s_{tt}]\!]$;
- $\{inch, cm\}$ is the direct content in $[\![attribute : 10, s_{tt}]\!]$;
- $\{e_{l.g.1}\}$ is the direct content in $[\![attribute : 10, s_{tt}]\!]$.

9.8. The content of attributes in the context of mediators

A set s_t is the content in $[\![attribute : a_{tt}, s_{tt}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}, mediator{-}degree : n_{at.1}]\!]$ if s_t is the set of all elements in $[\![attribute : a_{tt}, s_{tt}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}, mediator{-}degree : n_{at.1}]\!]$. A set s_t is the content in $[\![attribute : a_{tt}, c_{nf}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}, mediator{-}degree : n_{at.1}]\!]$ if s_t is the set of all elements in $[\![attribute : a_{tt}, c_{nf}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}, mediator{-}degree : n_{at.1}]\!]$.

A set s_t is the content in $[\![attribute : a_{tt}, s_{tt}, attribute{-}order : n_t, element{-}order : i_{nt}, mediator{-}degree : n_{at.1}]\!]$ if $s_t = \bigcup_{c_{ncpl}[\![s_{tt}]\!]} s_t[\![attribute : a_{tt}, s_{tt}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl}, mediator{-}degree : n_{at.1}]\!]$. A set s_t is the content in $[\![attribute : a_{tt}, c_{nf}, attribute{-}order : n_t, element{-}order : i_{nt}, mediator{-}degree : n_{at.1}]\!]$ if $s_t = \bigcup_{c_{ncpl.n}[\![c_{nf}]\!]} s_t$

$[\![attribute : a_{tt}, c_{nf}, attribute{-}order : n_t, element{-}order : i_{nt}, c_{ncpl.n}, mediator{-}degree : n_{at.1}]\!]$.

A set s_t is the content in $[\![attribute : a_{tt}, s_{tt}, attribute{-}order : n_t, mediator{-}degree : n_{at.1}]\!]$ if $s_t = \bigcup_{-n_t<i_{nt}} s_t[\![attribute : a_{tt}, s_{tt}, attribute{-}order : n_t, element{-}order : i_{nt}, mediator{-}degree : n_{at.1}]\!]$. A set s_t is the content in $[\![attribute : a_{tt}, c_{nf}, attribute{-}order : n_t, mediator{-}degree : n_{at.1}]\!]$ if $s_t = \bigcup_{-n_t<i_{nt}} s_t[\![attribute : a_{tt}, c_{nf}, attribute{-}order : n_t, element{-}order : i_{nt}, mediator{-}degree : n_{at.1}]\!]$.

A set s_t is the content in $[\![attribute : a_{tt}, s_{tt}, mediator{-}degree : n_{at.1}]\!]$ if $s_t = \bigcup_{n_t} s_t[\![attribute : a_{tt}, s_{tt}, attribute{-}order : i_{nt}, mediator{-}degree : n_{at.1}]\!]$. A set s_t is the content in $[\![attribute : a_{tt}, c_{nf}, mediator{-}degree : n_{at.1}]\!]$ if $s_t = \bigcup_{n_t} s_t[\![attribute : a_{tt}, c_{nf}, attribute{-}order : i_{nt}, mediator{-}degree : n_{at.1}]\!]$.

$\bigoplus$ Let $c_{ncl.1} = (-3 : 10, -2 : inch, -1 : area, 0 : e_{l.g.1}, 1 : triangle, 2 : Euclidean, 3 : 2)$, $c_{ncl.2} = (-3 : 10, -2 : inch, -1 : perimeter, 0 : e_{l.g.2}, 1 : triangle, 2 : Euclidean, 3 : 2)$, $c_{ncl.3} = (-3 : 10, -2 : inch, 0 : e_{l.g.3}, 1 : rectangle, 2 : Euclidean, 3 : 2)$, and $[support\ s_{tt}] = \{c_{ncl.1}, c_{ncl.2}, c_{ncl.3}\}$. Then the following properties hold:

- $\{e_{l.g.1}, e_{l.g.2}\}$ is the content in $[\![attribute : 10, s_{tt}, mediator{-}degree : 2]\!]$;
- $\{e_{l.g.3}\}$ is the content in $[\![attribute : 10, s_{tt}, mediator{-}degree : 1]\!]$;
- $\{triangle\}$ is the content in $[\![attribute : 10, s_{tt}, mediator{-}degree : 3]\!]$;
- $\{rectangle\}$ is the content in $[\![attribute : 10, s_{tt}, mediator{-}degree : 2]\!]$.

10. Classification and interpretation of attributes

Attributes are classified according to their orders.

10.1. Attributes of the order 1

An attribute a_{tt} in $[\![s_{tt}, 1]\!]$ models a usual attribute in $[\![s_{s.q.i}]\!]$. Elements in $[\![attribute : a_{tt}, s_{tt}, attribute{-}order : 1]\!]$ are individuals and concepts in $[\![s_{tt}]\!]$.

$\bigoplus$ Let $c_{ncl.1} = (-3 : 10, -2 : inch, -1 : area, 0 : e_{l.g.1}, 1 : triangle, 2 : Euclidean, 3 : 2)$, $c_{ncl.2} = (-3 : 10, -2 : inch, -1 : area, 0 : e_{l.g.2}, 1 : square, 2 : Riemannian, 3 : 3)$, and $[support\ s_{tt}] = \{c_{ncl.1}, c_{ncl.2}\}$. Then the following properties hold:

- the direct attribute *area* classifies geometric figures having area in $[\![s_{tt}]\!]$;
- the individuals $e_{l.g.1}$ and $e_{l.g.2}$ are elements of the order 0 of the direct attribute *area* in $[\![s_{tt}]\!]$ that means that classification of numerical characteristics of $e_{l.g.1}$ and $e_{l.g.2}$ includes area in $[\![s_{tt}]\!]$;

– the concepts *triangle* and *square* are elements of the order 1 of the direct attribute *area* in $[\![s_{tt}]\!]$ that means that classification of numerical characteristics of triangles and squares includes area in $[\![s_{tt}]\!]$;
– the concept spaces *Euclidean* and *Riemannian* are elements of the order 2 of the direct attribute *area* in $[\![s_{tt}]\!]$ that means that classification of numerical characteristics of geometric figures in Euclidean and Riemannian spaces includes area in $[\![s_{tt}]\!]$;
– the concept space spaces 2 and 3 are elements of the order 3 of the direct attribute *area* in $[\![s_{tt}]\!]$ that means that classification of numerical characteristics of geometric figures in two-dimensional and three-dimensional spaces includes area in $[\![s_{tt}]\!]$.

10.2. Attributes of the order 2

An attribute a_{tt} in $[\![s_{tt}, 2]\!]$ models an attribute space in $[\![s_{s.q.i}]\!]$. Elements in $[\![attribute : a_{tt}, s_{tt}, attribute{-}order : 2]\!]$ are direct attributes, individuals and concepts in $[\![s_{tt}]\!]$.

⊕ Let $c_{ncl.1} = (-3 : 10, -2 : inch, -1 : area, 0 : e_{l.g.1}, 1 : triangle, 2 : Euclidean, 3 : 2)$, $c_{ncl.2} = (-3 : 10, -2 : inch, -1 : perimeter, 0 : e_{l.g.2}, 1 : square, 2 : Riemannian, 3 : 3)$, and $[support\ s_{tt}] = \{c_{ncl.1}, c_{ncl.2}\}$. Then the following properties hold:
– the attribute space *inch* classifies numerical characteristics of geometric figures measured in inches in $[\![s_{tt}]\!]$;
– the direct attributes *area* and *perimeter* are elements of the order −1 of the attribute space *inch* in $[\![s_{tt}]\!]$ that means that classification of numerical characteristics of geometric figures measured in inches includes area and perimeter in $[\![s_{tt}]\!]$;
– the individuals $e_{l.g.1}$ and $e_{l.g.2}$ are elements of the order 0 of the attribute space *inch* in $[\![s_{tt}]\!]$ that means that classifications of geometric figures with numerical characteristics measured in inches includes $e_{l.g.1}$ and $e_{l.g.2}$ in $[\![s_{tt}]\!]$;
– the concepts *triangle* and *square* are elements of the order 1 of the attribute space *inch* in $[\![s_{tt}]\!]$ that means that classifications of geometric figures with numerical characteristics measured in inches includes triangles and squares $[\![s_{tt}]\!]$;
– the concept spaces *Euclidean* and *Riemannian* are elements of the order 2 of the attribute space *inch* in $[\![s_{tt}]\!]$ that means that classifications of spaces containing geometric figures with numerical characteristics measured in inches includes Euclidean and Riemannian spaces in $[\![s_{tt}]\!]$;
– the concept space spaces 2 and 3 are elements of the order 3 of the attribute space *inch*

in $[\![s_{tt}]\!]$ that means that classifications of dimensions of spaces containing geometric figures with numerical characteristics measured in inches includes dimensions 2 and 3 in $[\![s_{tt}]\!]$.

10.3. Attributes of the order 3

An attribute a_{tt} in $[\![s_{tt}, 3]\!]$ models a space of attribute spaces in $[\![s_{s.q.i}]\!]$. Elements in $[\![attribute : a_{tt}, s_{tt}, attribute{-}order : 3]\!]$ are attribute spaces, direct attributes, individuals and concepts in $[\![s_{tt}]\!]$.

⊕ Let $c_{ncl.1} = (-3 : 10, -2 : inch, -1 : area, 0 : e_{l.g.1}, 1 : triangle, 2 : Euclidean, 3 : 2)$, $c_{ncl.2} = (-3 : 10, -2 : cm, -1 : perimeter, 0 : e_{l.g.2}, 1 : square, 2 : Riemannian, 3 : 3)$, and $[support\ s_{tt}] = \{c_{ncl.1}, c_{ncl.2}\}$. Then the following properties hold:

- the attribute space space 10 classifies numerical characteristics of geometric figures with values represented in decimal system;
- the attribute spaces *inch* and *cm* are elements of the order -2 of the attribute space space 10 in $[\![s_{tt}]\!]$ that means that classifications of units of measurement of numerical characteristics of geometric figures with values represented in decimal system includes inches and centimeters in $[\![s_{tt}]\!]$;
- the direct attributes *area* and *perimeter* are elements of the order -11 of the attribute space space 10 in $[\![s_{tt}]\!]$ that means that classifications of numerical characteristics of geometric figures with values represented in decimal system includes area and perimeter in $[\![s_{tt}]\!]$;
- the individuals $e_{l.g.1}$ and $e_{l.g.2}$ are elements of the order 0 of the attribute space space 10 in $[\![s_{tt}]\!]$ that means that classifications of geometric figures with numerical characteristics with values represented in decimal system includes $e_{l.g.1}$ and $e_{l.g.2}$ in $[\![s_{tt}]\!]$;
- the concepts *triangle* and *square* are elements of the order 1 of the attribute space space 10 in $[\![s_{tt}]\!]$ that means that classifications of geometric figures with numerical characteristics with values represented in decimal system includes triangles and squares in $[\![s_{tt}]\!]$;
- the concept spaces *Euclidean* and *Riemannian* are elements of the order 2 of the attribute space space 10 in $[\![s_{tt}]\!]$ that means that classifications of spaces containing geometric figures with numerical characteristics with values represented in decimal

system includes Euclidean space and Riemannian space in $[\![s_{tt}]\!]$;

– the concept space spaces 10 and 2 are elements of the order 3 of the attribute space space 10 in $[\![s_{tt}]\!]$ that means that classifications of dimensions of spaces containing geometric figures with numerical characteristics with values represented in decimal system includes dimensions 10 and 2 in $[\![s_{tt}]\!]$.

10.4. Attributes of higher orders

An attribute a_{tt} in $[\![s_{tt}, n_t]\!]$, where $n_t > 3$, is classified and interpreted in the similar way (by the introduction of spaces of attribute space spaces and so on.).

11. Classification of conceptuals

11.1. General principles and definitions

We use the two-level scheme of classification of conceptuals. The upper (first) level is defined by the maximal order of attributes of a conceptual. This level is described by the notion of concretization order of a conceptual. The lower (second) level is defined by the set of all element orders of a conceptual. This level is described by the notion of integral order of a conceptual.

11.1.1. Concretization orders of conceptuals

The number 0 is an order in $[\![c_{ncpl}]\!]$ if the minimal order in $[\![c_{ncpl}, element :]\!]$ is greater than or equal to 0. A number n_t is an order in $[\![c_{ncpl}]\!]$ if $-n_t$ is a minimal order in $[\![c_{ncpl}, element :]\!]$.

⊕ Let $c_{ncl.1} = (-3 : 10, -2 : inch, -1 : area, 0 : e_{l.g.1}, 1 : triangle, 2 : Euclidean, 3 : 2)$, $c_{ncl.2} = (-2 : inch, -1 : area, 0 : e_{l.g.2}, 1 : triangle, 2 : Euclidean, 3 : 2)$, $c_{ncl.3} = (-1 : area, 0 : e_{l.g.3}, 1 : triangle, 2 : Euclidean, 3 : 2)$, $c_{ncl.4} = (0 : e_{l.g.4}, 1 : triangle, 2 : Euclidean, 3 : 2)$, $c_{ncl.5} = (1 : triangle, 2 : Euclidean, 3 : 2)$, $c_{ncl.6} = (2 : Euclidean, 3 : 2)$, and $c_{ncl.7} = (3 : 2)$. Then the conceptuals $c_{ncl.1}$, $c_{ncl.2}$, $c_{ncl.3}$ have the orders 3, 2, 1 and the conceptuals $c_{ncl.4}$, $c_{ncl.5}$, $c_{ncl.6}$, $c_{ncl.7}$ have the order 0.

Conceptuals of the order n_t concretizes conceptuals of the orders which are less than n_t. They define the special kinds of such conceptuals and are used to classify them. Concretization is performed by attributes of the order n_t and their values. Therefore, the order of a conceptual is also called the concretization order of the conceptual.

11.1.2. Integral orders of conceptuals

11.1.2.1. Integral orders

A set s_t is an integral order in $[\![c_{ncpl}]\!]$ if s_t is a set of all orders in $[\![c_{ncpl}, element :]\!]$.

⨁ Let $c_{ncl.1} = (-3 : 10, -2 : inch, -1 : area, 0 : e_{l.g.1}, 1 : triangle, 2 : Euclidean, 3 : 2)$, $c_{ncl.1} = (-3 : 10, -1 : area, 1 : triangle, 3 : 2)$, $c_{ncl.1} = (-2 : inch, -1 : area, 2 : Euclidean, 3 : 2)$. Then $o_{r.i}[\![c_{ncl.1}]\!] = \{-3, -2, -1, 0, 1, 2, 3\}$, $o_{r.i}[\![c_{ncl.2}]\!] = \{-3, -1, 1, 3\}$, and $o_{r.i}[\![c_{ncl.3}]\!] = \{-2, -1, 2, 3\}$.

11.1.2.2. Refined integral orders

A set s_t is a refined integral order in $[\![c_{ncpl}]\!]$ if s_t is a result of replacement of zero or more orders i_{nt} in $[\![[\![c_{ncpl}, element :]\!]]\!]$ in the set $o_{r.i}[\![c_{ncpl}]\!]$ by objects $i_{nt} : [c_{ncpl}\ i_{nt}]$. A refined integral order in $[\![c_{ncpl}]\!]$ refines an integral order in $[\![c_{ncpl}]\!]$, providing information on some elements of c_{ncpl} with their orders. Let $c_{ncpl} : o_{r.i.r}$ denote a conceptual c_{ncpl} which has the refined integral order $o_{r.i.r}$.

⨁ Let $c_{ncpl} = (-3 : 10, -2 : inch, -1 : area, 0 : e_{l.g.1}, 1 : triangle, 2 : Euclidean, 3 : 2)$. Then $\{-3, -2, -1, 0, 1, 2, 3\}$, $\{-3, -2 : inch, -1, 0, 1 : triangle, 2, 3\}$ and $\{-3 : 10, -2 : inch, -1 : area, 0 : e_{l.g.1}, 1 : triangle, 2 : Euclidean, 3 : 2\}$ are refined integral orders in $[\![c_{ncpl}]\!]$.

11.1.2.3. Properties of integral orders

Proposition 8. A conceptual c_{ncpl} has the single integral order.

Proof. This follows from the definition of the integral order of a conceptual. □

Proposition 9. A conceptual c_{ncpl} has a finite set of refined integral orders.

Proof. This follows from the definition of the refined integral order and the finite number of orders of conceptuals in the context of elements. □

Proposition 10. The integral order in $[\![c_{ncpl}]\!]$ is a refined integral order in $[\![c_{ncpl}]\!]$.

Proof. This follows from the definition of the refined integral order of a conceptual. □

11.1.2.4. Notes

Conceptuals of the same concretization order are classified according to their integral orders. Each integral order defines a separate kind of conceptuals.

Conceptuals allow to model ontological elements in detail. Each kind of conceptuals models a separate kind of ontological elements.

11.2. Modelling of ontological elements by conceptuals of the order 0

In this section conceptuals of the order 0 is classified according to their integral orders and the ontological elements modelled by conceptuals of this classification is described.

A conceptual $c_{ncpl} : \{0\}$ models the individual $[c_{ncpl}\ 0]$.

$\bigoplus$ The conceptual $(0 : f_g)$ models the geometric figure f_g.

A conceptual $c_{ncpl} : \{0, 1\}$ models the individual $[c_{ncpl}\ 0]$ from the concept $[c_{ncpl}\ 1]$.

$\bigoplus$ The conceptual $(0 : f_g, 1 : triangle)$ models the triangle f_g.

A conceptual $c_{ncpl} : \{1\}$ models the concept $[c_{ncpl}\ 1]$.

$\bigoplus$ A conceptual $(1 : triangle)$ models triangles.

A conceptual $c_{ncpl} : \{1, 2\}$ models the concept $[c_{ncpl}\ 1]$ from the concept space $[c_{ncpl}\ 2]$.

$\bigoplus$ The conceptual $(1 : triangle, 2 : Euclidean)$ models triangles in Euclidean space.

A conceptual $c_{ncpl} : \{2\}$ models the concept space $[c_{ncpl}\ 2]$.

$\bigoplus$ The conceptual $(2 : Euclidean)$ models Euclidean space.

A conceptual $c_{ncpl} : \{0, 2\}$ models the individual $[c_{ncpl}\ 0]$ from the concept space $[c_{ncpl}\ 2]$.

$\bigoplus$ The conceptual $(0 : f_g, 2 : Euclidean)$ models the geometric figure f_g in Euclidean space.

A conceptual $c_{ncpl} : \{0, 1, 2\}$ models the individual $[c_{ncpl}\ 0]$ from the concept $[c_{ncpl}\ 1]$ from the concept space $[c_{ncpl}\ 2]$.

$\bigoplus$ The conceptual $(0 : f_g, 1 : triangle, 2 : Euclidean)$ models the triangle f_g in Euclidean space.

Classification of other conceptuals of the order 0 and description of the ontological elements modelled by these conceptuals is performed in a similar way (by the introduction of the concept space space and so on.). For example, a conceptual $c_{ncpl} : \{0, 1, 2, 3\}$ models the individual $[c_{ncpl}\ 0]$ from the concept $[c_{ncpl}\ 1]$ from the concept space $[c_{ncpl}\ 2]$ from the concept space space $[c_{ncpl}\ 3]$.

$\bigoplus$ The conceptual $(0 : f_g, 1 : triangle, 2 : Euclidean, 3 : 2)$ models the triangle f_g in two-dimensional Euclidean space.

11.3. Modelling of ontological elements by conceptuals of the order 1

In this section conceptuals of the order 1 is classified according to their integral orders and the ontological elements modelled by conceptuals of this classification is described.

A conceptual $c_{ncpl} : \{-1\}$ models the attribute $[c_{ncpl}\ -1]$.

$\bigoplus$ The conceptual $(-1 : area)$ models area of geometric figures.

A conceptual $c_{ncpl} : \{-1, 0\}$ models the attribute $[c_{ncpl}\ -1]$ of the individual $[c_{ncpl}\ 0]$.

$\bigoplus$ The conceptual $(-1 : area, 0 : f_g)$ models area of the geometric figure f_g.

A conceptual $c_{ncpl} : \{-1, 0, 1\}$ models the attribute $[c_{ncpl}\ -1]$ of the individual $[c_{ncpl}\ 0]$ from the concept $[c_{ncpl}\ 1]$.

$\bigoplus$ The conceptual $(-1 : area, 0 : f_g, 1 : triangle)$ models area of the triangle f_g.

A conceptual $c_{ncpl} : \{-1, 1\}$ models the attribute $[c_{ncpl}\ -1]$ of individuals from the concept $[c_{ncpl}\ 1]$.

$\bigoplus$ The conceptual $(-1 : area, 1 : triangle)$ models area of triangles.

A conceptual $c_{ncpl} : \{-1, 0, 1, 2\}$ models the attribute $[c_{ncpl}\ -1]$ of the individual $[c_{ncpl}\ 0]$ from the concept $[c_{ncpl}\ 1]$ from the concept space $[c_{ncpl}\ 2]$.

$\bigoplus$ The conceptual $(-1 : area, 0 : f_g, 1 : triangle, 2 : Euclidean)$ models area of the triangle f_g in Euclidean space.

A conceptual $c_{ncpl} : \{-1, 1, 2\}$ models the attribute $[c_{ncpl}\ -1]$ of individuals from the concept $[c_{ncpl}\ 1]$ from the concept space $[c_{ncpl}\ 2]$.

$\bigoplus$ The conceptual $(-1 : area, 1 : triangle, 2 : Euclidean)$ models area of triangles in Euclidean space.

A conceptual $c_{ncpl} : \{-1, 0, 2\}$ models the attribute $[c_{ncpl}\ -1]$ of the individual $[c_{ncpl}\ 0]$ from the concept space $[c_{ncpl}\ 2]$.

$\bigoplus$ The conceptual $(-1 : area, 0 : f_g, 2 : Euclidean)$ models area of the geometric figure f_g in Euclidean space.

A conceptual $c_{ncpl} : \{-1, 2\}$ models the attribute $[c_{ncpl}\ -1]$ of individuals from concepts from the concept space $[c_{ncpl}\ 2]$.

$\bigoplus$ The conceptual $(-1 : area, 2 : Euclidean)$ models area of geometric figures in Euclidean space.

Correlation between other kinds of conceptuals of the order 1 and the corresponding kinds of ontological elements is performed in a similar way.

11.4. Modelling of ontological elements by conceptuals of the order 2

In this section conceptuals of the order 2 is classified according to their integral orders and the ontological elements modelled by conceptuals of this classification is described.

A conceptual $c_{ncpl} : \{-2, -1\}$ models the attribute $[c_{ncpl}\ -1]$ in the attribute space $[c_{ncpl}\ -2]$.

$\bigoplus$ The conceptual $(-2 : inch, -1 : area)$ models area measured in inches.

A conceptual $c_{ncpl} : \{-2, -1, 0\}$ models the attribute $[c_{ncpl}\ -1]$ of the individual $[c_{ncpl}\ 0]$ in the attribute space $[c_{ncpl}\ -2]$.

$\bigoplus$ The conceptual $(-2 : inch, -1 : area, 0 : f_g)$ models area of the geometric figure f_g measured in inches.

A conceptual $c_{ncpl} : \{-2, -1, 0, 1\}$ models the attribute $[c_{ncpl}\ -1]$ of the individual $[c_{ncpl}\ 0]$ from the concept $[c_{ncpl}\ 1]$ in the attribute space $[c_{ncpl}\ -2]$.

$\bigoplus$ The conceptual $(-2 : inch, -1 : area, 0 : f_g, 1 : triangle)$ models area of the triangle f_g measured in inches.

A conceptual $c_{ncpl} : \{-2, -1, 1\}$ models the attribute $[c_{ncpl}\ -1]$ of individuals from the concept $[c_{ncpl}\ 1]$ in the attribute space $[c_{ncpl}\ -2]$.

$\bigoplus$ The conceptual $(-2 : inch, -1 : area, 1 : triangle)$ models area of triangles measured in inches.

A conceptual $c_{ncpl} : \{-2, -1, 0, 1, 2\}$ models the attribute $[c_{ncpl}\ -1]$ of the individual $[c_{ncpl}\ 0]$ from the concept $[c_{ncpl}\ 1]$ from the concept space $[c_{ncpl}\ 2]$ in the attribute space $[c_{ncpl}\ -2]$.

$\bigoplus$ The conceptual $(-2 : inch, -1 : area, 0 : f_g, 1 : triangle, 2 : Euclidean)$ models area of the triangle f_g in Euclidean space measured in inches.

A conceptual $c_{ncpl} : \{-2, -1, 1, 2\}$ models the attribute $[c_{ncpl}\ -1]$ of individuals from the concept $[c_{ncpl}\ 1]$ from the concept space $[c_{ncpl}\ 2]$ in the attribute space $[c_{ncpl}\ -2]$.

$\bigoplus$ The conceptual $(-2 : inch, -1 : area, 1 : triangle, 2 : Euclidean)$ models area of triangles in Euclidean space measured in inches.

A conceptual $c_{ncpl} : \{-2, -1, 0, 2\}$ models the attribute $[c_{ncpl}\ -1]$ of the individual $[c_{ncpl}\ 0]$ from the concept space $[c_{ncpl}\ 2]$ in the attribute space $[c_{ncpl}\ -2]$.

$\bigoplus$ The conceptual $(-2 : inch, -1 : area, 0 : f_g, 2 : Euclidean)$ models area of the geometric figure f_g in Euclidean space measured in inches.

A conceptual $c_{ncpl} : \{-2, -1, 2\}$ models the attribute $[c_{ncpl}\ -1]$ of individuals from concepts from the concept space $[c_{ncpl}\ 2]$ in the attribute space $[c_{ncpl}\ -2]$.

$\bigoplus$ The conceptual $(-2 : inch, -1 : area, 2 : Euclidean)$ models area of geometric figures in Euclidean space measured in inches.

A conceptual $c_{ncpl} : \{-2, 0\}$ models the individual $[c_{ncpl}\ 0]$ in the attribute space $[c_{ncpl}\ -2]$.

⨁ The conceptual $(-2 : inch, 0 : f_g)$ models the geometric figure f_g with numerical characteristics measured in inches.

A conceptual $c_{ncpl} : \{-2, 0, 1\}$ models the individual $[c_{ncpl}\ 0]$ from the concept $[c_{ncpl}\ 1]$ in the attribute space $[c_{ncpl}\ -2]$.

⨁ The conceptual $(-2 : inch, 0 : f_g, 1 : triangle)$ models the triangle f_g with numerical characteristics measured in inches.

A conceptual $c_{ncpl} : \{-2, 1\}$ models the concept $[c_{ncpl}\ 1]$ in the attribute space $[c_{ncpl}\ -2]$.

⨁ The conceptual $(-2 : inch, 1 : triangle)$ models triangles with numerical characteristics measured in inches.

A conceptual $c_{ncpl} : \{-2, 1, 2\}$ models the concept $[c_{ncpl}\ 1]$ from the concept space $[c_{ncpl}\ 2]$ in the attribute space $[c_{ncpl}\ -2]$.

⨁ The conceptual $(-2 : inch, 1 : triangle, 2 : Euclidean)$ models triangles in Euclidean space with numerical characteristics measured in inches.

A conceptual $c_{ncpl} : \{-2, 2\}$ models the concept space $[c_{ncpl}\ 2]$ in the attribute space $[c_{ncpl}\ -2]$.

⨁ The conceptual $(-2 : inch, 2 : Euclidean)$ models geometric figures in Euclidean space with numerical characteristics measured in inches.

A conceptual $c_{ncpl} : \{-2, 0, 2\}$ models the individual $[c_{ncpl}\ 0]$ from the concept space $[c_{ncpl}\ 2]$ in the attribute space $[c_{ncpl}\ -2]$.

⨁ The conceptual $(-2 : inch, 0 : f_g, 2 : Euclidean)$ models the geometric figure f_g in Euclidean space with numerical characteristics measured in inches.

A conceptual $c_{ncpl} : \{-2, 0, 1, 2\}$ models the individual $[c_{ncpl}\ 0]$ from the concept $[c_{ncpl}\ 1]$ from the concept space $[c_{ncpl}\ 2]$ in the attribute space $[c_{ncpl}\ -2]$.

⨁ The conceptual $(-2 : inch, 0 : f_g, 1 : triangle, 2 : Euclidean)$ models the triangle f_g in Euclidean space with numerical characteristics measured in inches.

Correlation between other kinds of conceptuals of the order 2 and the corresponding kinds of ontological elements is performed in a similar way.

11.5. Modelling of ontological elements by conceptuals of the higher orders

Classification of conceptuals of the order 3 or higher and description of the ontological elements modelled by conceptuals of this classification is performed in a similar way (by the introduction of the attribute space space and so on.).

$\bigoplus$ The conceptual $(-3 : 10, -2 : inch, -1 : area, 0 : f_g, 1 : triangle, 2 : Euclidean, 3 : 2)$ models area of the triangle f_g in two-dimensional Euclidean space measured in inches in decimal system.

12. Modelling of relations, types, domains, inheritance

12.1. Relations and their instances

Finite binary relations are modelled by direct concepts and their instances are modelled by the elements of the order 0 of these concepts, represented by pairs of elements.

Finite relations of the arity n_t are modelled by direct concepts and their instances are modelled by the elements of the order 0 of these concepts, represented by sequence elements of the length n_t.

Finite relations of the variable arity are modelled by direct concepts and their instances are modelled by the elements of the order 0 of these concepts, represented by sequence elements of the variable length.

12.2. Types and domains

Finite types are modelled by direct concepts and their values are modelled by the elements of the order 0 of these concepts. Domains as the special kind of finite types are also modelled by direct concepts and their values are modelled by the elements of the order 0 of these concepts.

Types of attributes of the order n_t are modelled by the special attribute *type* of the order $n_t + 1$. Values of this attribute are types.

$\bigoplus$ Let $c_{ncpl} = (-2 : type, -1 : area, 0 : f_g)$, and $s_{tt} = (c_{ncpl} : real)$. Then the area of the geometric figure f_g is a real number in $[\![s_{tt}]\!]$.

$\bigoplus$ Let $c_{ncpl} = (-2 : type, -1 : area, 0 : *)$, and $s_{tt} = (c_{ncpl} : real)$. Then the area of any geometric figure is a real number in $[\![s_{tt}]\!]$. The semantics of $*$ is defined in section **??**

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12.3. Inheritance

12.3.1. Inheritance on elements

The usual inheritance relation on concepts is generalized to the inheritance relation on elements of the same order in $[\![s_{tt}]\!]$. It is modelled by the special direct concept *inheritance* and their instances are modelled by the elements of the order 0 of the concept *inheritance*, represented by the triples of elements. Elements of the triple specify the inheriting element, the inherited element and their order. An element e_l inherits from $e_{l.1}$ in $[\![s_{tt}, i_{nt}]\!]$ if $[s_{tt}\ (0 : (e_l, e_{l.1}, i_{nt}), 1 : inheritance)] \neq und$.

Inheritance on elements redefines interpretation *value* of conceptuals as follows:

- if $[s_{tt}\ c_{ncpl}] \neq und$, then $[value\ c_{ncpl}\ s_{tt}] = [s_{tt}\ c_{ncpl}]$;
- if $[s_{tt}\ c_{ncpl}] = und$, i_{nt} is a maximal order in $[\![c_{ncpl}, element :]\!]$, s_t is a set of $e_l[\![s_{tt}]\!]$ such that $[c_{ncpl}\ i_{nt}]$ inherits from e_l in $[\![s_{tt}, i_{nt}]\!]$, $s_t \neq \emptyset$, and $[value\ [c_{ncpl}\ i_{nt} : e_l]\ s_{tt}] = [value\ [c_{ncpl}\ i_{nt} : e_{l.1}]\ s_{tt}]$ for all $e_l, e_{l.1} \in s_t$, then $[value\ c_{ncpl}\ s_{tt}] = [value\ [c_{ncpl}\ i_{nt} : e_l]\ s_{tt}]$, where $e_l \in s_t$;
- otherwise, $[value\ c_{ncpl}\ s_{tt}] = und$.

12.3.2. Inheritance on direct concepts

The inheritance on direct concepts is the special case of the inheritance on elements.

A concept $c_{ncp.d}$ inherits from a concept $c_{o..pt.d.1}$ in $[\![s_{tt}]\!]$ if $c_{ncp.d}$ inherits from $c_{o..pt.d.1}$ in $[\![s_{tt}, 1]\!]$.

12.3.3. Inheritance on element sequences

The inheritance relation on elements is generalized to the inheritance relation on element sequences. This relation is modelled by the special direct concept *inheritance* :: *sq* and their instances are modelled by the elements of the order 0 of this concept, represented by the triples of sequence elements of the same length. The elements of the triple specify inheriting elements, inherited elements and their orders. An element $e_{l.(*)}$ inherits from $e_{l.(*).1}$ in $[\![s_{tt}, i_{nt.(*)}]\!]$ if $i_{nt.(*)} = (i_{nt.1}, ..., i_{nt.n_t})$, $i_{nt.1} < \ldots < i_{nt.n_t}$, $[len\ e_{l.(*)}] = [len\ e_{l.(*).1}] = n_t$, and $[s_{tt}\ (0 : (e_{l.(*)}, e_{l.(*).1}, i_{nt.(*)}), 1 : inheritance :: sq)] \neq und$.

Inheritance on ordered elements redefines interpretation *value* of conceptuals as follows:

- if $[s_{tt}\ c_{ncpl}] \neq und$, then $[value\ c_{ncpl}\ s_{tt}] = [s_{tt}\ c_{ncpl}]$;
- if
  - $[s_{tt}\ c_{ncpl}] = und$,
  - $i_{nt.1} < \ldots < i_{nt.n_t}$ are orders in $[\![c_{ncpl}, element :]\!]$,
  - for all i_{nt} if $i_{nt} \geq i_{nt.1}$ and i_{nt} is an order in $[\![c_{ncpl}, element :]\!]$, then i_{nt} coincides with one of the numbers $i_{nt.1}$, ..., $i_{nt.n_t}$,

– s_t is a set of $e_l[\![s_{tt}]\!]$ such that $([c_{ncpl}\ i_{nt.1}], \ldots, [c_{ncpl}\ i_{nt.n_t}])$ inherits from e_l in $[\![s_{tt}, (i_{nt.1}, \ldots, i_{nt.n_t})]\!]$,

– $s_t \neq \emptyset$,

– $[value\ [c_{ncpl}\ i_{nt.1} : [e_l\ .\ 1], \ldots, i_{nt.n_t} : [e_l\ .\ n_t]]\ s_{tt}] = [value\ [c_{ncpl}\ i_{nt.1} : [e_{l.1}\ .\ 1], \ldots, i_{nt.n_t} : [e_{l.1}\ .\ n_t]]\ s_{tt}]$ for each $e_l, e_{l.1} \in s_t$,

then $[value\ c_{ncpl}\ s_{tt}] = [value\ [c_{ncpl}\ i_{nt.1} : [e_l\ .\ 1], \ldots, i_{nt.n_t} : [e_l\ .\ n_t]]\ s_{tt}]$, where $e_l \in s_t$;

- otherwise, $[value\ c_{ncpl}\ s_{tt}] = und$.

13. Generic conceptuals

A generic conceptual defines a set of conceptuals satisfying a certain template and sets the default value for these conceptuals. Conceptuals from this set are called instances of the generic conceptual. The template of the generic conceptual is defined by its form.

13.1. The main definitions

13.1.1. Generic conceptuals

Let $* \in A_{tm}$. A conceptual $c_{ncpl}[\![s_{tt}]\!]$ is a generic conceptual in $[\![s_{tt}]\!]$ if there exists $o_{rd}[\![c_{ncpl}]\!]$ such that $[c_{ncpl}\ o_{rd}] \in \{*, (*, t_p), (*, t_p, p_{rm}), (*, *, p_{rm})\}$. The element $p_{l.s}$ of the form $[c_{ncpl}\ o_{rd}]$ from this definition is called a substitution place in $[\![c_{ncpl}, s_{tt}, o_{rd}]\!]$. The number o_{rd} is called an order in $[\![p_{l.s}, c_{ncpl}, s_{tt}]\!]$. The elements t_p and p_{rm} are called a type and parameter in $[\![p_{l.s}, c_{ncpl}, s_{tt}, o_{rd}]\!]$.

13.1.2. Kinds of generic conceptuals

A conceptual $c_{ncpl.g}$ is partially typed in $[\![s_{tt}]\!]$ if there exist $p_{l.s}$, t_p and o_{rd} such that $p_{l.s}$ is a substitution place in $[\![c_{ncpl.g}, s_{tt}, o_{rd}]\!]$ and t_p is a type in $[\![p_{l.s}, c_{ncpl.g}, s_{tt}, o_{rd}]\!]$.

A conceptual $c_{ncpl.g}$ is typed in $[\![s_{tt}]\!]$ if for all $p_{l.s}$ and o_{rd} if $p_{l.s}$ is a substitution place in $[\![c_{ncpl.g}, s_{tt}, o_{rd}]\!]$, then there exists t_p such that t_p is a type in $[\![p_{l.s}, c_{ncpl.g}, s_{tt}, o_{rd}]\!]$.

A conceptual $c_{ncpl.g}$ is parametric in $[\![s_{tt}]\!]$ if there exist $p_{l.s}$, p_{rm} and o_{rd} such that $p_{l.s}$ is a substitution place in $[\![c_{ncpl.g}, s_{tt}, o_{rd}]\!]$ and p_{rm} is a parameter in $[\![p_{l.s}, c_{ncpl.g}, s_{tt}, o_{rd}]\!]$.

13.1.3. Instances of generic conceptuals

A conceptual c_{ncpl} is an instance in $[\![c_{ncpl.g}, s_{tt}]\!]$, if the following properties hold:

- if $[c_{ncpl.g}\ i_{nt}]$ is not a substitution place in $[\![c_{ncpl.g}, s_{tt}, i_{nt}]\!]$, then $[c_{ncpl}\ i_{nt}] = [c_{ncpl.g}\ i_{nt}]$;

- if $[c_{ncpl.g}\ i_{nt}]$ is a substitution place in $[\![c_{ncpl.g}, s_{tt}, i_{nt}]\!]$, then $[c_{ncpl}\ i_{nt}]$ is an element in $[\![s_{tt}, i_{nt}]\!]$;
- if $[c_{ncpl.g}\ i_{nt}] \in \{(*, t_p), (*, t_p, p_{rm})\}$, then $[c_{ncpl}\ i_{nt}]$ is an element in $[\![concept : t_p, s_{tt}, concept{-}order : 1, element{-}order : 0]\!]$;
- if p_{rm} is a parameter in $[\![p_{l.s.1}, c_{ncpl.g}, s_{tt}, o_{r.e.1}]\!]$ and $[\![p_{l.s.2}, c_{ncpl.g}, s_{tt}, o_{r.e.2}]\!]$, then $[c_{ncpl}\ o_{r.e.1}] = [c_{ncpl}\ o_{r.e.2}]$.

13.1.4. States with generic conceptuals

A state s_{tt} is a state with generic conceptuals, if the following properties hold:

- *(the consistency property)* if $c_{ncl.g.1} \neq c_{ncl.g.2}$, then there is no c_{ncpl} such that c_{ncpl} is an instance of $c_{ncl.g.1}$ in $[\![s_{tt}]\!]$ and c_{ncpl} is an instance of $c_{ncl.g.2}$ in $[\![s_{tt}]\!]$;
- interpretation *value* of conceptuals is redefined as follows:
  - if $[s_{tt}\ c_{ncpl}] \neq und$, then $[value\ c_{ncpl}\ s_{tt}] = [s_{tt}\ c_{ncpl}]$;
  - if $[s_{tt}\ c_{ncpl}] = und$ and c_{ncpl} is an instance in $[\![c_{ncpl.g}, s_{tt}]\!]$, then $[value\ c_{ncpl}\ s_{tt}] = [s_{tt}\ c_{ncpl.g}]$;
  - otherwise, $[value\ c_{ncpl}\ s_{tt}] = und$.

13.2. Examples of generic conceptuals

A conceptual $c_{ncpl.g} : \{-1, 0 : *, 1\}$ models the property that the value of the attribute $[c_{ncpl.g}\ -1]$ of individuals from the concept $[c_{ncpl.g}\ 1]$ equals $[s_{tt}\ c_{ncpl.g}]$ in $[\![s_{tt}]\!]$ if it is not defined explicitly.

⨁ The conceptual $c_{ncpl.g} = (-1 : area, 0 : *, 1 : triangle)$ models the property that area of triangles equals $[s_{tt}\ c_{ncpl.g}]$ in $[\![s_{tt}]\!]$ if it is not defined explicitly.

A conceptual $c_{ncpl.g} : \{-1, 0 : *\}$ models the property that the value of the attribute $[c_{ncpl.g}\ -1]$ of individuals equals $[s_{tt}\ c_{ncpl.g}]$ in $[\![s_{tt}]\!]$ if it is not defined explicitly.

⨁ The conceptual $c_{ncpl.g} = (-1 : area, 0 : *)$ models the property that area of geometric figures equals $[s_{tt}\ c_{ncpl.g}]$ in $[\![s_{tt}]\!]$ if it is not defined explicitly.

A conceptual $c_{ncpl.g} : \{0 : *, 1\}$ models the property that the value of individuals from the concept $[c_{ncpl.g}\ 1]$ equals $[s_{tt}\ c_{ncpl.g}]$ in $[\![s_{tt}]\!]$ if it is not defined explicitly.

⨁ The conceptual $c_{ncpl.g} = (0 : *, 1 : triangle)$ models the property that the value of triangles equals $[s_{tt}\ c_{ncpl.g}]$ in $[\![s_{tt}]\!]$ if it is not defined explicitly. What is the value of a triangle depends on interpretation.

13.3. Modelling of ontological elements and their properties based on generic conceptuals

Generic conceptuals together with attributes allow to model ontological elements and their properties in more detail.

A conceptual $c_{ncpl.g} : \{-2 : type, -1, 0 : *, 1\}$ models the property that the type of the attribute $[c_{ncpl.g}\ -1]$ of individuals from the concept $[c_{ncpl.g}\ 1]$ equals $[s_{tt}\ c_{ncpl.g}]$ in $[\![s_{tt}]\!]$ if it is not defined for individuals explicitly.

⊕ The conceptual $c_{ncpl.g} = (-2 : type, -1 : area, 0 : *, 1 : triangle)$ models the property that the type of the attribute $area$ of triangles equals $[s_{tt}\ c_{ncpl.g}]$ in $[\![s_{tt}]\!]$ if it is not defined for triangles explicitly.

A conceptual $c_{ncpl.g} : \{-2 : type, -1, 0 : *\}$ models the property that the type of the attribute $[c_{ncpl.g}\ -1]$ of individuals equals $[s_{tt}\ c_{ncpl.g}]$ in $[\![s_{tt}]\!]$ if it is not defined for individuals explicitly.

⊕ The conceptual $c_{ncpl.g} = (-2 : type, -1 : area, 0 : *)$ models the property that the type of the attribute $area$ of geometric figures equals $[s_{tt}\ c_{ncpl.g}]$ in $[\![s_{tt}]\!]$ if it is not defined for geometric figures explicitly.

A conceptual $c_{ncpl.g} : \{-2 : type, 0 : *\}$ models the property that the type of individuals equals $[s_{tt}\ c_{ncpl.g}]$ in $[\![s_{tt}]\!]$ if it is not defined for individuals explicitly.

⊕ The conceptual $c_{ncpl.g} = (-2 : type, 0 : *)$ models the property that the type of geometric figures equals $[s_{tt}\ c_{ncpl.g}]$ in $[\![s_{tt}]\!]$ if it is not defined for geometric figures explicitly.

A conceptual $c_{ncpl.g} : \{-2 : type, 0 : *, 1\}$ models the property that the type of individuals from the concept $[c_{ncpl.g}\ 1]$ equals $[s_{tt}\ c_{ncpl.g}]$ in $[\![s_{tt}]\!]$ if it is not defined for such individuals explicitly.

⊕ The conceptual $c_{ncpl.g} = (-2 : type, 0 : *, 1 : triangle)$ models the property that the type of triangles equals $[s_{tt}\ c_{ncpl.g}]$ in $[\![s_{tt}]\!]$ if it is not defined for triangles explicitly.

14. The CCSL language

The CCSL language (Conceptual Configuration System Language) is a basic language of CCSs. Interpretable elements of CCSL are called basic elements of CCSs.

Let $s_b \subseteq (x : x_0,\ y : y_0,\ z : z_0,\ u : u_0,\ v : v_0,\ w : w_0,\ x_1 : x_{1.0},\ ...,\ x_{n_t} : x_{n_t.0},\ conf :: in : c_{nf})$.

14.1. Syntax of CCSL

An object o_b is an atom in CCSL if

- o_b is a sequence of Unicode symbols except for the whitespace symbols and the symbols ", ′, (,), ;, ,, and :, or
- o_b is a special atom, or
- o_b has the form "$o_{b.1}$" called a string, where $o_{b.1}$ is a sequence of Unicode symbols in which each occurrence of the symbol " is preceded by the symbol ′ and each occurrence of the symbol ′ is doubled.

The set $A_{to.s}$ of special atoms includes the object ::= and can be extended.

An object o_b is an element in CCSL if $o_b \in A_{tm}$, $o_b = e_l : e_{l.1}$, $o_b = (e_{l.*})$, or $o_b = e_l :: e_{l.1}$.

The whitespace symbols and the semicolon in CCSL are element delimiters along with comma. For example, $(e_{l.1},\ e_{l.2})$, $(e_{l.1};\ e_{l.2})$ and $(e_{l.1}\ e_{l.2})$ represent the same element.

An element $e_{l.a}$ is a conceptual in CCSL if all its attributes are integers.

An element $e_{l.a}$ is a conceptual state in CCSL if all its attributes are conceptuals.

An element $e_{l.a}$ is a conceptual configuration in CCSL if $[image\ e_{l.a}] \subseteq S_{tt}$.

The element $(pattern\ p_t\ var\ (v_{r.*})\ seq\ (v_{r.s.*}))$ in CCSL represents the pattern specification $(p_t, (v_{r.*}), (v_{r.s.*}))$.

The element $(definition\ p_t\ var\ (v_{r.*})\ seq\ (v_{r.s.*})\ then\ b_d) :: name :: n_m$ in CCSL represents the element definition $(p_t, (v_{r.*}), (v_{r.s.*}), b_d)$ with the name n_m.

For simplicity, we omit the names of interpretations and definitions below.

14.2. The special forms for interpretations and definitions

In this section we define the special forms for interpretations and definitions used below.

The form $(interpretation\ p_t\ var\ (v_{r.*})\ seq\ (v_{r.s.*})\ then\ f_n) :: name :: n_m$ denotes the interpretation $(p_t, (v_{r.*}), (v_{r.s.*}), f_n)$ with the name n_m.

The objects $var\ (v_{r.*})$ and $seq\ (v_{r.s.*})$ in the form $(interpretation\ ...)$ can be omitted. The omitted objects correspond to var () and seq (), respectively.

Let $\{v_{r.*}\}$, $\{v_{r.s.*}\}$, $\{v_{r.*.1}\}$ and $\{v_{r.*.2}\}$ are pairwise disjoint, and $\{v_{r.*.3}\} \subseteq \{v_{r.*}\} \cup \{v_{r.*.1}\} \cup \{v_{r.*.2}\}$. The form $(definition\ p_t\ var\ (v_{r.*})\ seq\ (v_{r.s.*})\ abn\ (v_{r.*.1})\ und\ (v_{r.*.2})\ val\ (v_{r.*.3})\ where\ c_{nd}\ then\ b_d)$ called a definition form is defined as follows:

- $(definition\ p_t\ var\ (v_{r.*})\ seq\ (v_{r.s.*})\ und\ (v_{r.*.1})\ abn\ (v_{r.*.2})\ val\ (v_{r.*.3})\ where\ c_{nd}\ then\ b_d)$ is a shortcut for $(definition\ p_t\ var\ (v_{r.*})\ seq\ (v_{r.s.*})\ abn\ (v_{r.*.1})\ und\ (v_{r.*.2})\ val\ (v_{r.*.3})\ then\ (if\ c_{nd}\ then\ b_d\ else\ und))$;

- $(definition\ p_t\ var\ (v_{r.*})\ seq\ (v_{r.s.*})\ und\ (v_{r.*.1})\ abn\ (v_{r.*.2})\ val\ (v_{r.*.3},\ v_r)\ then\ b_d)$ is a shortcut for $(definition\ p_t\ var\ (v_{r.*})\ seq\ (v_{r.s.*})\ und\ (v_{r.*.1})\ abn\ (v_{r.*.2})\ val\ (v_{r.*.3})\ then\ (let\ w\ be\ v_r\ in\ [subst\ (v_r :: * : w)\ b_d]))$, where w is a new element that does not occur in this definition;
- $(definition\ p_t\ var\ (v_{r.*})\ seq\ (v_{r.s.*})\ und\ (v_{r.*.1})\ abn\ (v_{r.*.2})\ val\ ()\ then\ b_d)$ is a shortcut for $(definition\ p_t\ var\ (v_{r.*})\ seq\ (v_{r.s.*})\ und\ (v_{r.*.1})\ abn\ (v_{r.*.2})\ then\ b_d)$;
- $(definition\ p_t\ var\ (v_{r.*})\ seq\ (v_{r.s.*})\ und\ (v_{r.*.1},\ v_r)\ abn\ (v_{r.*.2})\ then\ b_d)$ is a shortcut for $(definition\ p_t\ var\ (v_{r.*})\ seq\ (v_{r.s.*})\ und\ (v_{r.*.1})\ abn\ (v_{r.*.2})\ then\ (if\ (v_r\ is\ undefined)\ then\ und\ else\ b_d))$;
- $(definition\ p_t\ var\ (v_{r.*})\ seq\ (v_{r.s.*})\ und\ ()\ abn\ (v_{r.*.2})\ then\ b_d)$ is a shortcut for $(definition\ p_t\ var\ (v_{r.*})\ seq\ (v_{r.s.*})\ abn\ (v_{r.*.2})\ then\ b_d)$;
- $(definition\ p_t\ var\ (v_{r.*})\ seq\ (v_{r.s.*})\ abn\ (v_{r.*.2},\ v_r)\ then\ b_d)$ is a shortcut for $(definition\ p_t\ var\ (v_{r.*})\ seq\ (v_{r.s.*})\ abn\ (v_{r.*.2})\ then\ (if\ (v_r\ is\ abnormal)\ then\ v_r\ else\ b_d))$;
- $(definition\ p_t\ var\ (v_{r.*})\ seq\ (v_{r.s.*})\ abn\ ()\ then\ b_d)$ is a shortcut for $(definition\ p_t\ var\ (v_{r.*})\ seq\ (v_{r.s.*})\ then\ b_d)$.

The element c_{nd} specifies the restriction on the values of the pattern variables. The undefined value is propagated through the variables of $v_{r.*.1}$. Abnormal values are propagated through the variables of $v_{r.*.2}$. The special element $v_r :: *$ references to the value of element associated with the pattern variable v_r. A pattern variable is evaluated if the element associated with it is evaluated. Thus, the sequence $v_{r.*.3}$ contains evaluated pattern variables. A pattern variable is quoted if the element associated with it is not evaluated. Let $F_{rm.d}$ be a set of definition forms.

The objects $var\ (v_{r.*})$, $seq\ (v_{r.s.*})$, $und\ (v_{r.*.1})$, $abn\ (v_{r.*.2})$, $val\ (v_{r.*.3})$ and $where\ c_{nd}$ in the form $(definition\ ...)$ can be omitted. The omitted objects correspond to $var\ ()$, $seq\ ()$, $und\ ()$, $abn\ ()$, $val\ ()$ and $where\ true$, respectively.

15. Semantics of interpretable elements in CCSL

15.1. Abnormal elements operations

The element und is defined as follows:

$(definition\ \ und\ \ then\ \ und :: q)$.

The element e_{xc} is defined as follows:

$(definition\ \ x\ \ var\ \ (x)\ \ where\ \ (x\ \ is\ \ exception)\ \ then\ \ x :: q) :: name :: (\text{"@"},\ \ exception)$.

The definition satisfies the property: $n_m \prec_{[\![o_{rd.intr}]\!]}$ ("@", $exception$) for each n_m such that n_m is a name of an atomic element interpretation or element definition with the pattern distinct from v_r, where v_r is a variable of this pattern.

The element $e_l :: q$ is defined as follows:

$(interpretation\ \ x :: q\ \ var\ \ (x)\ \ then\ \ f_n)$,

where $[f_n\ s_b] = x_0$.

15.2. Statements

The element $(if\ x\ then\ y\ else\ z)$ is defined as follows:

$(definition\ \ (if\ \ x\ \ then\ \ y\ \ else\ \ z)\ \ var\ \ (x,\ \ y,\ \ z)\ \ val\ \ (x)$

$\ then\ \ (if\ \ x :: *\ \ then\ \ y\ \ else\ \ z) :: atm)$;

$(interpretation\ \ (if\ \ x\ \ then\ \ y\ \ else\ \ z) :: atm\ \ var\ \ (x,\ \ y,\ \ z)\ \ then\ \ f_n)$,

where $[f_n\ s_b] = [if\ [x_0 \neq und]\ then\ [value\ y_0\ [s_b\ conf :: in]]\ else\ [value\ z_0\ [s_b\ conf :: in]]]$.

The element $(if\ x\ then\ y\ elseif\ z\ then\ u\ ...\ else\ v)$ is defined as follows:

$(definition\ \ (if\ \ x\ \ then\ \ y\ \ elseif\ \ z)\ \ var\ \ (x,\ \ y,\ \ z)\ \ seq\ \ (z)$

$\ then\ \ (if\ \ x\ \ then\ \ y\ \ else\ \ (if\ \ z)))$.

The element $(let\ x\ be\ y\ in\ z)$ is defined as follows:

$(interpretation\ \ (let\ \ x\ \ be\ \ y\ \ in\ \ z)\ \ var\ \ (x,\ \ y,\ \ z)\ \ then\ \ f_n)$,

where $[f_n\ s_b] = [value\ [subst\ (x_0 : [value\ y_0\ [s_b\ conf :: in]])\ z_0]\ [s_b\ conf :: in]]$.

The element e_l of the form $(let\ ::\ seq\ x\ be\ y\ in\ z)$, where $x \in E_{l.(*)}$, $y \in E_{l.(*)}$, and $[len\ x] = [len\ y]$, is defined by the rule

$(rule\ \ (let :: seq\ \ x,\ \ y\ \ be\ \ z,\ \ u\ \ in\ \ v)\ \ var\ \ (x,\ \ z,\ \ v)\ \ seq\ \ (y,\ \ u)$

$\ then\ \ (let\ \ x\ \ be\ \ z\ \ in\ \ (let :: seq\ \ y\ \ be\ \ u\ \ in\ \ v)))$;

$(rule\ \ (let :: seq\ \ be\ \ in\ \ v)\ \ var\ \ (v)\ \ then\ \ v)$.

The elements x, y and z are called a substitution variables specification, substitution values specification and substitution body in $[\![e_l]\!]$. The elements of x and y are called substitution variables and substitution values in $[\![e_l]\!]$.

15.3. Characteristic functions for defined concepts

An object $d_{f.c}$ is a concept definition if $d_{f.c}$ is an interpretation of the form $(interpretation$ $(e_{l.1}\ is\ e_{l.2})\ var\ (v_{r.*})\ seq\ (v_{r.s.*})\ then\ f_n) :: name :: n_m$, or $d_{f.c}$ is a definition of the form $(definition\ (e_{l.1}\ is\ e_{l.2})\ var\ (v_{r.*})\ seq\ (v_{r.s.*})\ then\ b_d) :: name :: n_m$. Concept definitions specify concepts and their instances. Concepts specified by them are called defined concepts. The elements $e_{l.1}$ and $e_{l.2}$ are called an instance pattern and concept pattern in $[\![d_{f.c}]\!]$. The element $(e_{l.1}\ is\ e_{l.2})$ is called a characteristic function in $[\![d_{f.c}]\!]$. Let $D_{f.c}$ be a set of concept definitions.

An element $c_{ncp.d}$ is a defined concept in $[\![d_{f.c}, s_b]\!]$ if c_{ncp} is an instance in $[\![(e_{l.2}, var\ (v_{r.*})\ seq\ (v_{r.s.*})), m_t, s_b]\!]$. An element $c_{ncp.d}$ is a defined concept in $[\![d_{f.c}]\!]$ if there exists s_b such that $c_{ncp.d}$ is a defined concept in $[\![d_{f.c}, s_b]\!]$. An element $c_{ncp.d}$ is a defined concept in $[\![c_{nf}]\!]$ if there exists $d_{f.c}[\![c_{nf}]\!]$ such that $c_{ncp.d}$ is a defined concept in $[\![d_{f.c}]\!]$. Let $C_{ncp.d}$ be a set of defined concepts.

An element i_{nstn} is an instance in $[\![d_{f.c}, s_b]\!]$ if i_{nstn} is an instance in $[\![(e_{l.1}, var\ (v_{r.*})\ seq\ (v_{r.s.*})), m_t, s_b]\!]$. An element i_{nstn} is an instance in $[\![d_{f.c}]\!]$ if there exists s_b such that $c_{ncp.d}$ is an instance in $[\![d_{f.c}, s_b]\!]$.

An element i_{nstn} is an instance in $[\![c_{ncp.d}, c_{nf}, d_{f.c}]\!]$ if i_{nstn} is an instance in $[\![d_{f.c}, c_{ncp.d}]\!]$ is a defined concept in $[\![d_{f.c}]\!]$, and $[value\ (i_{nstn}\ is\ c_{ncp.d})\ c_{nf}\ (n_m)] \neq und$. An element i_{nstn} is an instance in $[\![c_{ncp.d}, c_{nf}]\!]$ if there exists $d_{f.c}$ such that i_{nstn} is an instance in $[\![c_{ncp.d}, c_{nf}, d_{f.c}]\!]$. An element $c_{ncp.d}$ is an instance in $[\![c_{nf}, m_t]\!]$ if there exists $c_{ncp.d}$ such that i_{nstn} is an instance in $[\![c_{ncp.d}, c_{nf}]\!]$. Let I_{nstn} be a set of instances.

A set s_t is called a content in $[\![c_{ncp.d}, c_{nf}]\!]$ if s_t is a set of all i_{nstn} such that i_{nstn} is an instance in $[\![c_{ncp.d}, c_{nf}]\!]$. Let $[content\ c_{ncp.d}\ c_{nf}]$ denote the content in $[\![c_{ncp.d}, c_{nf}]\!]$.

The notion of defined concepts is extended to the definitions of the form $(definition\ (e_{l.1}\ is\ e_{l.2})\ var\ (v_{r.*})\ seq\ (v_{r.s.*})\ und\ (v_{r.*.1})\ val\ (v_{r.*.3})\ where\ c_{nd}\ then\ b_d)$. Let d_f have this form. An element $c_{ncp.d}$ is a defined concept in $[\![d_f, s_b]\!]$ if $c_{ncp.d}$ is a defined concept in $[\![d_{f.1}, s_b]\!]$, where $d_{f.1}$ is a definition of the form $(definition\ (e_{l.1}\ is\ e_{l.2})\ var\ (v_{r.*})\ seq\ (v_{r.s.*})\ then\ b_{d.1})$ such that d_f is reduced to $d_{f.1}$.

The element $(x\ is\ atom)$ specifying that x is an atom is defined as follows:

$(interpretation\ (x\ is\ atom)\ var\ (x)\ then\ f_n)$,

where $[f_n\ s_b] = [if\ [x_0 \in A_{tm}]\ then\ true\ else\ und]$.

The element $(x\ is\ update)$ specifying that x is an element update is defined as follows:

$(interpretation\ (x\ is\ update)\ var\ (x)\ then\ f_n)$,

where $[f_n\ s_b] = [if\ [x_0 \in U_{p.e}]\ then\ true\ else\ und]$.

The element $(x\ is\ multi{-}attribute)$ specifying that x is a multi-attribute element is defined as follows:

$(interpretation\ (x\ is\ multi{-}attribute)\ var\ (x)\ then\ f_n)$,

where $[f_n\ s_b] = [if\ [x_0 \in E_{l.ma}]\ then\ true\ else\ und]$.

The element $(x\ is\ attribute)$ specifying that x is an attribute element is defined as follows:

$(interpretation\ (x\ is\ attribute)\ var\ (x)\ then\ f_n)$,

where $[f_n\ s_b] = [if\ [x_0 \in E_{l.a}]\ then\ true\ else\ und]$.

The element $(x\ is\ sorted)$ specifying that x is a sorted element is defined as follows:

$(interpretation\ (x\ is\ sorted)\ var\ (x)\ then\ f_n)$,

where $[f_n\ s_b] = [if\ [x_0 \in E_{l.s}]\ then\ true\ else\ und]$.

The element $(x\ is\ undefined)$ specifying that x equals und is defined as follows:

$(interpretation\ (x\ is\ undefined)\ var\ (x)\ then\ f_n)$,

where $[f_n\ s_b] = [if\ [x_0 = und]\ then\ true\ else\ und]$.

The element $(x\ is\ defined)$ specifying that x does not equal und is defined as follows:

$(interpretation\ (x\ is\ defined)\ var\ (x)\ then\ f_n)$.

where $[f_n\ s_b] = [if\ [x_0 \neq und]\ then\ true\ else\ und]$.

The element $(x\ is\ exception)$ specifying that x is an exception is defined as follows:

$(interpretation\ (x\ is\ exception)\ var\ (x)\ then\ f_n)$,

where $[f_n\ s_b] = [if\ [x_0 \in E_{xc}]\ then\ true\ else\ und]$.

The element $(x\ is\ normal)$ specifying that x is a normal element is defined as follows:

$(interpretation\ (x\ is\ normal)\ var\ (x)\ then\ f_n)$,

where $[f_n\ s_b] = [if\ [x_0 \in E_{l.n}]\ then\ true\ else\ und]$.

The element $(x\ is\ abnormal)$ specifying that x is an abnormal element is defined as follows:

$(interpretation\ (x\ is\ abnormal)\ var\ (x)\ then\ f_n)$,

where $[f_n\ s_b] = [if\ [x_0 \in E_{l.ab}]\ then\ true\ else\ und]$.

The element $(x\ is\ sequence)$ specifying that x is a sequence element is defined as follows:

$(interpretation\ (x\ is\ sequence)\ var\ (x)\ then\ f_n)$,

where $[f_n\ s_b] = [if\ [x_0 \in E_{l.(*)}]\ then\ true\ else\ und]$.

The element $(x\ is\ set)$ specifying that the elements of the sequence element x are pairwise distinct is defined as follows:

$(definition\ (x\ is\ set)\ var\ (x)\ where\ (x\ is\ sequence)\ then\ (x\ is\ set) :: atm)$;

$(interpretation\ (x\ is\ set) :: atm\ var\ (x)\ then\ f_n)$,

where $[f_n\ s_b] = [if\ [[x_0\ .\ n_{t.1}] \neq [x_0\ .\ n_{t.2}]$ for all $n_{t.1}$ and $n_{t.2}$ such that $n_{t.1} \neq n_{t.2}, n_{t.1} \leq [len\ x_0]$ and $n_{t.2} \leq [len\ x_0]]\ then\ true\ else\ und]$.

The element $(x\ is\ empty)$ specifying that x is an empty element is defined as follows:

$(definition\ (x\ is\ empty)\ var\ (x)\ then\ (x :: q\ =\ ()))$.

The element (x *is nonempty*) specifying that x is not an empty element is defined as follows: $(definition\ \ (x\ \ is\ \ nonempty)\ \ var\ \ (x)\ \ then\ \ (x :: q\ \ != \ \ ()))$.

The element (x *is conceptual*) specifying that x is a conceptual is defined as follows:
$(interpretation\ \ (x\ \ is\ \ conceptual)\ \ var\ \ (x)\ \ then\ \ f_n)$,
where $[f_n\ s_b] = [if\ [x_0 \in C_{ncpl}]\ then\ true\ else\ und]$.

The element (x *is* (*conceptual in* y)) specifying that x is a conceptual in the context of the state y is defined as follows:
$(definition\ \ (x\ \ is\ \ (conceptual\ \ in\ \ y))\ \ var\ \ (x,\ \ y)$
$where\ \ ((x\ \ is\ \ conceptual)\ \ and\ \ (y\ \ is\ \ state))\ \ then\ \ (x\ \ is\ \ conceptual\ \ in\ \ y) :: atm)$;
$(interpretation\ \ (x\ \ is\ \ (conceptual\ \ in\ \ y)) :: atm\ \ var\ \ (x,\ \ y)\ \ then\ \ f_n)$,
where $[f_n\ s_b] = [if\ [x_0 \in C_{ncpl}[\![y_0]\!]]\ then\ true\ else\ und]$.

The element (x *is state*) specifying that x is a conceptual state is defined as follows:
$(interpretation\ \ (x\ \ is\ \ state)\ \ var\ \ (x)\ \ then\ \ f_n)$,
where $[f_n\ s_b] = [if\ [x_0 \in S_{tt}]\ then\ true\ else\ und]$.

The element (x *is configuration*) specifying that x is a conceptual configuration is defined as follows:
$(interpretation\ \ (x\ \ is\ \ configuration)\ \ var\ \ (x)\ \ then\ \ f_n)$,
where $[f_n\ s_b] = [if\ [x_0 \in C_{nf}]\ then\ true\ else\ und]$.

The element (x *is nat*) specifying that x is a natural number is defined as follows:
$(interpretation\ \ (x\ \ is\ \ nat)\ \ var\ \ (x)\ \ then\ \ f_n)$,
where $[f_n\ s_b] = [if\ [x_0 \in N_t]\ then\ true\ else\ und]$.

The element (x *is nat*0) specifying that x is either a natural number, or a zero is defined as follows:
$(interpretation\ \ (x\ \ is\ \ nat0)\ \ var\ \ (x)\ \ then\ \ f_n)$,
where $[f_n\ s_b] = [if\ [x_0 \in N_{t0}]\ then\ true\ else\ und]$.

The element (x *is int*) specifying that x is an integer is defined as follows:
$(interpretation\ \ (x\ \ is\ \ int)\ \ var\ \ (x)\ \ then\ \ f_n)$,
where $[f_n\ s_b] = [if\ [x_0 \in I_{nt}]\ then\ true\ else\ und]$.

The element (x *is* (*satisfiable in* y)) specifying that x is satisfiable in the context of variables y is defined as follows:
$(definition\ \ (x\ \ is\ \ (satisfiable\ \ in\ \ y))\ \ var\ \ (x,\ \ y)\ \ where\ \ (y\ \ is\ \ sequence)$
$then\ \ (x\ \ is\ \ (satisfiable\ \ in\ \ y)) :: atm)$;

$(interpretation\ \ (x\ \ is\ \ (satisfiable\ \ in\ \ y)) :: atm\ \ var\ \ (x,\ \ y)\ \ then\ \ f_n)$,

where $[f_n\ s_b] = [if\ [x_0$ is satisfiable in $[\![(y_0, [s_b\ conf :: in])]\!]]\ then\ true\ else\ und]$.

The element $(x\ is\ (valid\ in\ y))$ specifying that x is valid in the context of variables y is defined as follows:

$(definition\ \ (x\ \ is\ \ (valid\ \ in\ \ y))\ \ var\ \ (x,\ \ y)\ \ where\ \ (y\ \ is\ \ sequence)$

$then\ \ (x\ \ is\ \ (valid\ \ in\ \ y)) :: atm)$;

$(interpretation\ \ (x\ \ is\ \ (valid\ \ in\ \ y)) :: atm\ \ var\ \ (x,\ \ y)\ \ then\ \ f_n)$,

where $[f_n\ s_b] = [if\ [x_0$ is valid in $[\![(y_0, [s_b\ conf :: in])]\!]]\ then\ true\ else\ und]$.

The element $(x\ is\ (sequence\ y))$ specifying that x is a sequence element such that the value in $[\![(e_l\ is\ y)]\!]$ does not equal und for each element e_l of x is defined as follows:

$(definition\ \ ((x\ \ y)\ \ is\ \ (sequence\ \ z))\ \ var\ \ (x,\ \ z)\ \ seq\ \ (y)$

$then\ \ ((x\ \ is\ \ z)\ \ and\ \ ((y)\ \ is\ \ (sequence\ \ z)))$;

$(definition\ \ (()\ \ is\ \ (sequence\ \ x))\ \ var\ \ (x)\ \ then\ \ true)$.

15.4. Elements operations

The element () is defined as follows:

$(definition\ \ ()\ \ then\ \ () :: q)$.

The element $(len\ x)$ specifying the length of the element x is defined as follows:

$(definition\ \ (len\ \ x)\ \ var\ \ (x)\ \ val\ \ (x)\ \ then\ \ (len\ \ x :: *) :: atm)$;

$(interpretation\ \ (len\ \ x) :: atm\ \ var\ \ (x)\ \ then\ \ f_n)$,

where

- if $x_0 \in A_{tm} \cup U_{p.e} \cup E_{l.s}$, then $[f_n\ s_b] = 1$;
- if $x_0 = (e_{l.*})$, then $[f_n\ s_b] = [len\ e_{l.*}]$.

The element $(x\ = \ y)$ specifying the equality of the elements x and y is defined as follows:

$(definition\ \ (x\ \ =\ \ y)\ \ var\ \ (x,\ \ y)\ \ val\ \ (x,\ \ y)$

$then\ \ (x :: *\ \ =\ \ y :: *) :: atm)$;

$(interpretation\ \ (x\ \ =\ \ y) :: atm\ \ var\ \ (x,\ \ y)\ \ then\ \ f_n)$,

where

- if x_0 and y_0 are equal atoms, then $[f_n\ s_b] = true$;
- if $x_0 \in U_{p.e}$, $y_0 \in U_{p.e}$, $a_{rg}[\![x_0]\!] = a_{rg}[\![y_0]\!]$, and $v_l[\![x_0]\!] = v_l[\![y_0]\!]$, then $[f_n\ s_b] = true$;
- if $x_0 \in E_{l.s}$, $y_0 \in E_{l.s}$, $e_l[\![x_0]\!] = e_l[\![y_0]\!]$, and $s_{rt}[\![x_0]\!] = s_{rt}[\![y_0]\!]$, then $[f_n\ s_b] = true$;
- if $x_0 \in E_{l.(*)}$, $y_0 \in E_{l.(*)}$, and x_0 and y_0 are equal sequences, then $[f_n\ s_b] = true$;

- otherwise, $[f_n\ s_b] = und$.

The element $(x\ !=\ y)$ specifying the inequality of the elements x and y is defined in the similar way.

The element $(x\ .\ y)$ specifying the y-th element of the sequence element x is defined as follows:

$(definition\ (x\ .\ y)\ var\ (x,\ y)\ val\ (x,\ y)$
$\ where\ ((x :: *\ is\ sequence)\ and\ (y :: *\ is\ nat))\ then\ (x :: *\ .\ y :: *) :: atm)$;
$(interpretation\ (x\ .\ y) :: atm\ var\ (x,\ y)\ then\ f_n)$,
where $[f_n\ s_b] = [x_0\ .\ y_0]$.

The element $(x\ ..\ y)$ specifying the value of the attribute element x for the attribute y is defined as follows:

$(definition\ (x\ ..\ y)\ var\ (x,\ y)\ val\ (x)\ where\ (x :: *\ is\ attribute)$
$\ then\ (x :: *\ ..\ y) :: atm)$;
$(interpretation\ (x\ ..\ y) :: atm\ var\ (x,\ y)\ then\ f_n)$,
where $[f_n\ s_b] = [x_0\ y_0]$.

The element $(x\ +\ y)$ specifying the concatenation of the sequence elements x and y is defined as follows:

$(definition\ (x\ +\ y)\ var\ (x,\ y)\ val\ (x,\ y)$
$\ where\ ((x :: *\ is\ sequence)\ and\ (y :: *\ is\ sequence))\ then\ (x :: *\ +\ y :: *) :: atm)$;
$(interpretation\ (x\ +\ y) :: atm\ var\ (x,\ y)\ then\ f_n)$,
where $[f_n\ s_b] = (e_{l.*}\ e_{l.1.*})$ for some $e_{l.*}$ and $e_{l.1.*}$ such that $x_0 = (e_{l.*})$ and $y_0 = e_{l.1.*}$.

The element $(x\ .+\ y)$ specifying the addition of the element x to the head of the sequence element y is defined as follows:

$(definition\ (x\ .+\ y)\ var\ (x,\ y)\ val\ (x,\ y)\ where\ (y :: *\ is\ sequence)$
$\ then\ (x :: *\ .+\ y :: *) :: atm)$;
$(interpretation\ (x\ .+\ y) :: atm\ var\ (x,\ y)\ then\ f_n)$,
where $[f_n\ s_b] = [if\ [y_0 = (e_{l.*})$ for some $e_{l.*}]\ then\ (x_0\ e_{l.*})\ else\ und]$.

The element $(x\ .+ :: set\ y)$ specifying the addition of the element x to the head of the sequence element y representing a set is defined as follows:

$(definition\ (x\ .+ :: set\ y)\ var\ (x,\ y)\ val\ (x,\ y)\ where\ (y :: *\ is\ set)$
$\ then\ (x :: *\ .+ :: set\ y :: *) :: atm)$;
$(interpretation\ (x\ .+ :: set\ y) :: atm\ var\ (x,\ y)\ then\ f_n)$,

where $[f_n\ s_b] = [if\ [y_0 = (e_{l.*})$ for some $e_{l.*}]\ then\ [if\ [x_0 \in e_{l.*}]\ then\ (e_{l.*})\ else\ (x_0\ e_{l.*})]\ else\ und]$.

The element $(x\ \ +.\ y)$ specifying the addition of the element y to the tail of the sequence element x is defined as follows:

$(definition\ \ (x\ \ +.\ y)\ \ var\ \ (x,\ \ y)\ \ val\ \ (x,\ \ y)\ \ where\ \ (x :: *\ \ is\ \ sequence)$

$then\ \ (x :: *\ \ +.\ y :: *) :: atm);$

$(interpretation\ \ (x\ \ +.\ y) :: atm\ \ var\ \ (x,\ \ y)\ \ then\ \ f_n),$

where $[f_n\ s_b] = [if\ [x_0 = (e_{l.*})$ for some $e_{l.*}]\ then\ (e_{l.*}\ y_0)\ else\ und]$.

The element $(x\ \ +.::set\ y)$ specifying the addition of the element y to the tail of the sequence element x representing a set is defined as follows:

$(definition\ \ (x\ \ +.::set\ \ y)\ \ var\ \ (x,\ \ y)\ \ val\ \ (x,\ \ y)\ \ where\ \ (x :: *\ \ is\ \ set)$

$then\ \ (x :: *\ \ +.::set\ \ y :: *) :: atm);$

$(interpretation\ \ (x\ \ +.::set\ \ y) :: atm\ \ var\ \ (x,\ \ y)\ \ then\ \ f_n),$

where $[f_n\ s_b] = [if\ [x_0 = (e_{l.*})$ for some $e_{l.*}]\ then\ [if\ [y_0 \in e_{l.*}]\ then\ (e_{l.*})\ else\ (e_{l.*}\ y_0)]\ else\ und]$.

The element $(x\ \ -.::set\ y)$ specifying the deletion of the element y from the sequence element x representing a set is defined as follows:

$(definition\ \ (x\ \ -.::set\ \ y)\ \ var\ \ (x,\ \ y)\ \ val\ \ (x,\ \ y)\ \ where\ \ (x :: *\ \ is\ \ set)$

$then\ \ (x :: *\ \ -.::set\ \ y :: *) :: atm);$

$(interpretation\ \ (x\ \ -.::set\ \ y) :: atm\ \ var\ \ (x,\ \ y)\ \ then\ \ f_n),$

where $[f_n\ s_b] = [if\ [x_0 = (e_{l.*.1}\ y_0\ e_{l.*.2})$ for some $e_{l.*.1}$ and $e_{l.*.2}]\ then\ (e_{l.*.1}\ e_{l.*.2})\ else\ [if\ [x_0 = (e_{l.*})$ for some $e_{l.*}]\ then\ (e_{l.*})\ else\ und]]$.

The element $(upd\ x\ y_1 : z_1, ..., y_{n_t} : z_{n_t})$ specifying the sequential updates of the attribute element x at the points y_1, ..., y_{n_t} by z_1, ..., z_{n_t} is defined as follows:

$(definition\ \ (upd\ \ x\ \ y)\ \ var\ \ (x)\ \ seq\ \ (y)\ \ val\ \ (x)$

$where\ \ ((x :: *\ \ is\ \ attribute)\ \ and\ \ ((y)\ \ is\ \ (sequence\ \ update)))\ \ then\ \ (upd :: att\ \ x :: *\ \ y));$

$(definition\ \ (upd :: att\ \ x\ \ y\ \ z)\ \ var\ \ (y)\ \ seq\ \ (z)\ \ und\ \ (x)$

$then\ \ (let\ \ w\ \ be\ \ (upd1 :: att\ \ x\ \ y)\ \ in\ \ (upd :: att\ \ w\ \ z)));$

$(definition\ \ (upd :: att\ \ x)\ \ var\ \ (x)\ \ then\ \ x);$

$(definition\ \ (upd1 :: att\ \ x\ \ y : z)\ \ var\ \ (x,\ \ y,\ \ z)\ \ val\ \ (z)$

$then\ \ (upd1 :: att\ \ x\ \ y : z :: *) :: atm);$

$(interpretation\ \ (upd1 :: att\ \ x\ \ y : z) :: atm\ \ var\ \ (x,\ \ y,\ \ z)\ \ then\ \ f_n),$

where $[f_n\ s_b] = [x_0\ y_0 : z_0]$.

The element $(upd\ x\ y : z)$ specifying the update of the sequence element x at the index y by z is defined as follows:

$(definition\ \ (upd\ \ x\ \ y\ \ z)\ \ var\ \ (x,\ \ y,\ \ z)\ \ val\ \ (x,\ \ y,\ \ z)$

$where\ \ ((x :: *\ \ is\ \ sequence)\ \ and\ \ (y :: *\ \ is\ \ nat)\ \ and\ \ (y :: *\ \ <=\ \ ((len\ \ x :: * :: q)\ \ +\ \ 1)))$

$then\ \ (upd :: seq\ \ x :: *\ \ y :: *\ \ z :: *) :: atm)$;

$(interpretation\ \ (upd :: seq\ \ x\ \ y : z) :: atm\ \ var\ \ (x,\ \ y,\ \ z)\ \ then\ \ f_n)$,

where $[f_n\ s_b] = [att{-}obj{-}to{-}seq\ [[seq{-}to{-}att{-}obj\ x_0]\ y_0 : z_0]]$.

The element $(x\ in :: set\ y)$ specifying that x is an element of the sequence element y is defined as follows:

$(definition\ \ (x\ \ in :: set\ \ y)\ \ var\ \ (x,\ \ y)\ \ where\ \ (y\ \ is\ \ sequence)$

$then\ \ (x\ \ in :: set\ \ y) :: atm)$;

$(interpretation\ \ (x\ \ in :: set\ \ y) :: atm\ \ var\ \ (x,\ \ y)\ \ then\ \ f_n)$,

where $[f_n\ s_b] = [x_0 \in y_0]$.

The element $(x\ includes :: set\ y)$ specifying that the sequence element x includes the elements of the sequence element y is defined as follows:

$(definition\ \ (x\ \ includes :: set\ \ y)\ \ var\ \ (x,\ \ y)$

$where\ \ ((x\ \ is\ \ sequence)\ \ and\ \ (y\ \ is\ \ sequence))\ \ then\ \ (x\ \ includes :: set\ \ y) :: atm)$;

$(interpretation\ \ (x\ \ includes :: set\ \ y) :: atm\ \ var\ \ (x,\ \ y)\ \ then\ \ f_n)$,

where $[f_n\ s_b] = [if\ [e_l \in x_0 \text{ for each } e_l \in y_0]\ then\ true\ else\ und]$.

The element $(attributes\ in\ x)$ specifying the sequence of attributes of the attribute element x is defined as follows:

$(definition\ \ (attributes\ \ in\ \ x)\ \ var\ \ (x)\ \ where\ \ (x\ \ is\ \ attribute)$

$then\ \ (attributes\ \ in\ \ x) :: atm)$;

$(interpretation\ \ (attributes\ \ in\ \ x) :: atm\ \ var\ \ (x,\ \ y)\ \ then\ \ f_n)$,

where $[f_n\ s_b] = (a_{rg.1}, ..., a_{rg.nt0})$ for $x_0 = (a_{rg.1} : v_{l.1}, ..., a_{rg.nt0} : v_{l.nt0})$.

The element $(values\ in\ x)$ specifying the sequence of attribute values of the attribute element x is defined as follows:

$(definition\ \ (values\ \ in\ \ x)\ \ var\ \ (x)\ \ where\ \ (x\ \ is\ \ attribute)\ \ then\ \ (values\ \ in\ \ x) :: atm)$;

$(interpretation\ \ (values\ \ in\ \ x) :: atm\ \ var\ \ (x,\ \ y)\ \ then\ \ f_n)$,

where $[f_n\ s_b] = (v_{l.1}, ..., v_{l.nt0})$ for $x_0 = (a_{rg.1} : v_{l.1}, ..., a_{rg.nt0} : v_{l.nt0})$.

The element $(element\ in\ x)$ specifying the element of the sorted element x is defined as follows:

$(definition\ \ (element\ \ in\ \ x)\ \ var\ \ (x)\ \ then\ \ (if\ \ x\ \ matches\ \ y :: z\ \ var\ \ (y,\ \ z)\ \ then\ \ y :: q))$.

The element $(sort\ in\ x)$ specifying the sort of the sorted element x is defined as follows:

$(definition\ \ (sort\ \ in\ \ x)\ \ var\ \ (x)\ \ then\ \ (if\ \ x\ \ matches\ \ y :: z\ \ var\ \ (y,\ \ z)\ \ then\ \ z :: q))$.

The element $(attribute\ in\ x)$ specifying the attribute of the element update x is defined as follows:

$(definition\ \ (attribute\ \ in\ \ x)\ \ var\ \ (x)\ \ then\ \ (if\ \ x\ \ matches\ \ y : z\ \ var\ \ (y,\ \ z)\ \ then\ \ y :: q))$.

The element $(value\ in\ x)$ specifying the value of the element update x is defined as follows:

$(definition\ \ (value\ \ in\ \ x)\ \ var\ \ (x)\ \ then\ \ (if\ \ x\ \ matches\ \ y : z\ \ var\ \ (y,\ \ z)\ \ then\ \ z :: q))$.

15.5. Boolean operations

The element $true$ is defined as follows:

$(definition\ \ true\ \ then\ \ true :: q)$.

The element $(x\ and\ y)$ specifying the conjunction of x and y is defined as follows:

$(definition\ \ (x\ \ and\ \ y)\ \ var\ \ (x,\ \ y)\ \ then\ \ (if\ \ x\ \ then\ \ y\ \ else\ \ und))$.

The elements $(x\ o_p\ y)$, where $o_p \in \{or, =>, <=>\}$ specifying the disjunction, implication and equivalence of x and y are defined in the similar way.

The element $(x_1\ and\ x_2\ and\ ...\ and\ x_{n_t})$ specifying the conjunction of x_1, x_2, ..., x_{n_t} is defined as follows:

$(definition\ \ (x\ \ and\ \ y\ \ and\ \ z)\ \ var\ \ (x,\ \ y)\ \ seq\ \ (z)\ \ then\ \ ((x\ \ and\ \ y)\ \ and\ \ z)$.

The element $(x_1\ or\ x_2\ or\ ...\ or\ x_{n_t})$ specifying the disjunction of x_1, x_2, ..., x_{n_t} is defined in the similar way.

The element $(not\ x)$ specifying the negation of x is defined as follows:

$(definition\ \ (not\ \ x)\ \ var\ \ (x)\ \ then\ \ (if\ \ x\ \ then\ \ und\ \ else\ \ true))$.

15.6. Integers

The element i_{nt} is defined as follows:

$(definition\ \ x\ \ var\ \ (x)\ \ where\ \ (x\ \ is\ \ int)\ \ then\ \ x :: q) :: name :: (\text{"@"},\ \ int)$.

The definition satisfies the property: $(\text{"@"}, exception) \prec_{[\![o_{rd.intr}]\!]} (\text{"@"}, int)$.

The element $(x\ +\ y)$ specifying the sum of x and y is defined as follows:

$(definition\ \ (x\ \ +\ \ y)\ \ var\ \ (x,\ \ y)\ \ val\ \ (x,\ \ y)$

$where\ \ ((x :: *\ \ is\ \ int)\ \ and\ \ (y :: *\ \ is\ \ int))\ \ then\ \ (x :: *\ \ +\ \ y :: *) :: atm)$;

$(interpretation\ \ (x\ \ +\ \ y) :: atm\ \ var\ \ (x,\ \ y)\ \ then\ \ f_n)$, where $[f_n\ s_b] = [x_0\ +\ y_0]$.

The elements $(x\ o_p\ y)$, where $o_p \in \{-, *, \}$, specifying the integer operations $-$ and $*$ are defined in the similar way.

The element $(x\ div\ y)$ specifying the quotient of x divided by y is defined as follows:

$(definition\ \ (x\ \ div\ \ y)\ \ var\ \ (x,\ \ y)\ \ val\ \ (x,\ \ y)$

$\ where\ \ ((x :: *\ \ is\ \ int)\ \ and\ \ (y :: *\ \ is\ \ int)\ \ and\ \ (y :: *\ \ ! =\ \ 0))$

$\ then\ \ (x :: *\ \ div\ \ y :: *) :: atm)$;

$(interpretation\ \ (x\ \ div\ \ y) :: atm\ \ var\ \ (x,\ \ y)\ \ then\ \ f_n)$,

where $[f_n\ s_b] = [x_0\ div\ y_0]$.

The element $(x\ mod\ y)$ specifying the integer operation mod is defined in the similar way.

The element $(x\ <\ y)$ specifying that x is less than y is defined as follows:

$(definition\ \ (x\ \ <\ \ y)\ \ var\ \ (x,\ \ y)\ \ val\ \ (x,\ \ y)$

$\ where\ \ ((x :: *\ \ is\ \ int)\ \ and\ \ (y :: *\ \ is\ \ int))\ \ then\ \ (x :: *\ \ <\ \ y :: *) :: atm)$;

$(interpretation\ \ (x\ \ <\ \ y) :: atm\ \ var\ \ (x,\ \ y)\ \ then\ \ f_n)$,

where $[f_n\ s_b] = [x_0\ <\ y_0]$.

The elements $(x\ o_p\ y)$, where $o_p \in \{<=, >, >=\}$, specifying the integer relations $\leq$, $>$ and $\geq$, are defined in the similar way.

15.7. Conceptuals operations

The element $(x\ in\ y)$ specifying the value of the conceptual x in the state y is defined as follows:

$(definition\ \ (x\ \ in\ \ y)\ \ var\ \ (x,\ \ y)$

$\ where\ \ ((x\ \ is\ \ conceptual)\ \ and\ \ (z\ \ is\ \ state))\ \ then\ \ (x\ \ in\ \ y) :: atm)$;

$(interpretation\ \ (x\ \ in\ \ y) :: atm\ \ var\ \ (x,\ \ y)\ \ then\ \ f_n)$,

where $[f_n\ s_b] = [y_0\ x_0]$.

The element $x :: state :: y$ specifying the value of the conceptual x in the substate with the name y of the current configuration is defined as follows:

$(definition\ \ x :: state :: y\ \ var\ \ (x,\ \ y)\ \ where\ \ (x\ \ is\ \ conceptual)$

$\ then\ \ (x\ \ in\ \ (conf :: q\ \ ..\ \ y))\ \ x :: state :: y :: atm)$;

$(in\ \ x :: state :: y :: atm\ \ var\ \ (x,\ \ y)\ \ then\ \ f_n)$,

where $(x_0 :: state :: y_0 :: atm, e_{l.*}\ \#\ c_{nf} \rightarrow_{f_n, s_b} e_{l.*}\ \#\ [[c_{nf}\ y_0]\ x_0]\ \#\ c_{nf}$.

The element c_{ncpl} is a shortcut for $c_{ncpl} :: ()$.

15.8. Countable concepts operations

A normal element $c_{ncp.c}$ is a countable concept in $[\![c_{nf}]\!]$ if $[[c_{nf}\ countable{-}concept]\ (0 : c_{ncp.c})] \in N_t$. Thus, the substate *countable−concept* specifies countable concepts. Let $C_{ncp.c}$ be a set of countable concepts. The element $[[c_{nf}\ countable{-}concept]\ (0 : c_{ncp.c})]$ is called an order in $[\![c_{ncp.c}, c_{nf}]\!]$. Let $O_{rd.cncp.c}$ be a set of orders of countable concepts. An element $n_t :: cc :: c_{ncp.c}$ is called an instance in $[\![c_{ncp.c}]\!]$. An element $n_t :: cc :: c_{ncp.c}$ is an instance in $[\![c_{ncp.c}, c_{nf}]\!]$ if $n_t \leq o_{rd.cncp.c}[\![c_{ncp.c}, c_{nf}]\!]$.

The element $(x\ is\ countable{-}concept)$ specifying that x is a countable concept is defined as follows:

$(definition\ \ (x\ \ is\ \ countable{-}concept)\ \ var\ \ (x)$
$\ \ then\ \ (let\ \ w\ \ be\ \ ((cnf\ \ ..\ \ countable{-}concept)\ \ ..\ \ (0 : x))\ \ in\ \ (w\ \ is\ \ int))$.

The element $n_t :: cc :: c_{ncp.c}$ is defined by the rule:

$(definition\ \ x :: cc :: y\ \ var\ \ (x,\ \ y)\ \ where\ \ ((x\ \ is\ \ int)\ \ and\ \ (y\ \ is\ \ countable{-}concept))$
$\ \ then\ \ x :: cc :: y :: q)$.

15.9. Matching operations

The conditional pattern matching element e_l of the form $(if\ x\ matches\ y\ var\ z\ seq\ u\ then\ v\ else\ w)$, where (y, z, u) is a pattern specification, is defined as follows:

$(definition\ \ (if\ \ x\ \ matches\ \ y\ \ var\ \ z\ \ seq\ \ u\ \ then\ \ v\ \ else\ \ w)\ \ var\ \ (x,\ \ y,\ \ z,\ \ u,\ \ v,\ \ w)$
$\ \ where\ \ ((z\ \ is\ \ sequence)\ \ and\ \ (u\ \ is\ \ sequence)\ \ and\ \ (z\ \ includes :: set\ \ u))$
$\ \ then\ \ (if\ \ x\ \ matches\ \ y\ \ var\ \ z\ \ seq\ \ u\ \ then\ \ v\ \ else\ \ w) :: atm)$;
$(interpretation\ \ (if\ \ x\ \ matches\ \ y\ \ var\ \ z\ \ seq\ \ u\ \ then\ \ v\ \ else\ \ w) :: atm$
$\ \ var\ \ (x,\ \ y,\ \ z,\ \ u,\ \ v,\ \ w)\ \ then\ \ f_n)$,

where $[value\ (if\ x_0\ matches\ y_0\ var\ z_0\ seq\ u_0\ then\ v_0\ else\ w_0) :: atm\ s_b\ c_{nf}],\ e_{l.*}\ \#\ c_{nf} \rightarrow_{f_n,s_b}$ $[if\ [x_0$ is an instance in $[\![(y_0, z_0, u_0), m_t,\ s_{b.1}]\!]$ for some $s_{b.1}]\ then\ [subst\ s_{b.1} \cup (conf :: in : c_{nf})\ v_0]\ else\ [subst\ (conf :: in : c_{nf})\ w_0],\ e_{l.*}\ \#\ c_{nf}$. The objects x, y, z, u, v and w are called a matched element, pattern, variable specification, sequence variable specification, *then*-branch and *else*-branch in $[\![e_l]\!]$. The elements of z are called pattern variables in $[\![e_l]\!]$. The element e_l executes the instance of the *then*-branch v in $[\![s_{b.1}]\!]$ if x is an instance in $[\![y, s_{b.1}]\!]$. Otherwise, the element e_l executes the *else*-branch w.

Let $\{v_{r.*}\}$, $\{v_{r.s.*}\}$, $\{v_{r.*.1}\}$ and $\{v_{r.*.2}\}$ are pairwise disjoint, and $\{v_{r.*.3}\} \subseteq \{v_{r.*}\} \cup \{v_{r.*.1}\} \cup \{v_{r.*.2}\}$. The form $(if\ e_l\ matches\ p_t\ var\ (v_{r.*})\ seq\ (v_{r.s.*})\ abn\ (v_{r.*.1})\ und\ (v_{r.*.2})\ val\ (v_{r.*.3})\ where$

c_{nd} *then* $e_{l.1}$ *else* $e_{l.2}$) is defined as follows:

- (if e_l *matches* p_t *var* ($v_{r.*}$) *seq* ($v_{r.s.*}$) *und* ($v_{r.*.1}$) *abn* ($v_{r.*.2}$) *val* ($v_{r.*.3}$) *where* c_{nd} *then* $e_{l.1}$ *else* $e_{l.2}$) is a shortcut for (if e_l *matches* p_t *var* ($v_{r.*}$) *seq* ($v_{r.s.*}$) *abn* ($v_{r.*.1}$) *und* ($v_{r.*.2}$) *val* ($v_{r.*.3}$) *then* (if c_{nd} *then* $e_{l.1}$ *else* $e_{l.2}$ :: (*nosubstexcept conf* :: *in*)) *else* $e_{l.2}$);
- (if e_l *matches* p_t *var* ($v_{r.*}$) *seq* ($v_{r.s.*}$) *und* ($v_{r.*.1}$) *abn* ($v_{r.*.2}$) *val* ($v_{r.*.3}$, v_r) *then* $e_{l.1}$ *else* $e_{l.2}$) is a shortcut for (if e_l *matches* p_t *var* ($v_{r.*}$) *seq* ($v_{r.s.*}$) *und* ($v_{r.*.1}$) *abn* ($v_{r.*.2}$) *val* ($v_{r.*.3}$) *then* (*let* w *be* v_r *in* [*subst* (v_r :: $*$: w) $e_{l.1}$]) *else* $e_{l.2}$), where w is a new element that does not occur in this definition;
- (if e_l *matches* p_t *var* ($v_{r.*}$) *seq* ($v_{r.s.*}$) *und* ($v_{r.*.1}$) *abn* ($v_{r.*.2}$) *val* () *then* $e_{l.1}$ *else* $e_{l.2}$) is a shortcut for (if e_l *matches* p_t *var* ($v_{r.*}$) *seq* ($v_{r.s.*}$) *und* ($v_{r.*.1}$) *abn* ($v_{r.*.2}$) *then* $e_{l.1}$ *else* $e_{l.2}$);
- (if e_l *matches* p_t *var* ($v_{r.*}$) *seq* ($v_{r.s.*}$) *und* ($v_{r.*.1}$, v_r) *abn* ($v_{r.*.2}$) *then* b_d) is a shortcut for (if e_l *matches* p_t *var* ($v_{r.*}$) *seq* ($v_{r.s.*}$) *und* ($v_{r.*.1}$) *abn* ($v_{r.*.2}$) *then* (if (v_r *is undefined*) *then und else* $e_{l.1}$) *else* $e_{l.2}$);
- (if e_l *matches* p_t *var* ($v_{r.*}$) *seq* ($v_{r.s.*}$) *und* () *abn* ($v_{r.*.2}$) *then* $e_{l.1}$ *else* $e_{l.2}$) is a shortcut for (if e_l *matches* p_t *var* ($v_{r.*}$) *seq* ($v_{r.s.*}$) *abn* ($v_{r.*.2}$) *then* $e_{l.1}$ *else* $e_{l.2}$);
- (if e_l *matches* p_t *var* ($v_{r.*}$) *seq* ($v_{r.s.*}$) *abn* ($v_{r.*.2}$, v_r) *then* $e_{l.1}$ *else* $e_{l.2}$) is a shortcut for (if e_l *matches* p_t *var* ($v_{r.*}$) *seq* ($v_{r.s.*}$) *abn* ($v_{r.*.2}$) *then* (if (v_r *is abnormal*) *then* v_r *else* $e_{l.1}$) *else* $e_{l.2}$);
- (if e_l *matches* p_t *var* ($v_{r.*}$) *seq* ($v_{r.s.*}$) *abn* () *then* $e_{l.1}$ *else* $e_{l.2}$) is a shortcut for (if e_l *matches* p_t *var* ($v_{r.*}$) *seq* ($v_{r.s.*}$) *then* $e_{l.1}$ *else* $e_{l.2}$).

The element c_{nd} specifies the restriction on the values of the pattern variables. The undefined value is propagated through the variables of $v_{r.*.1}$. Abnormal values are propagated through the variables of $v_{r.*.2}$. The special element $v_r :: *$ references to the value of element associated with the pattern variable v_r. A pattern variable is evaluated if the element associated with it is evaluated. Thus, the sequence $v_{r.*.3}$ contains evaluated pattern variables. A pattern variable is quoted if the element associated with it is not evaluated.

The objects *var* ($v_{r.*}$), *seq* ($v_{r.s.*}$), *und* ($v_{r.*.1}$), *abn* ($v_{r.*.2}$), *val* ($v_{r.*.3}$), *where* c_{nd} and *else* $e_{l.2}$ in this form can be omitted. The omitted objects correspond to *var* (), *seq* (), *und* (), *abn* (), *val* (), *where true* and *else skip*, respectively.

The form (e_l *matches* p_t *var* ($v_{r.*}$) *seq* ($v_{r.s.*}$) *und* ($v_{r.*.1}$) *abn* ($v_{r.*.2}$) *val* ($v_{r.*.3}$) *where* c_{nd}) is a shortcut for (if e_l *matches* p_t *var* ($v_{r.*}$) *seq* ($v_{r.s.*}$) *und* ($v_{r.*.1}$) *abn* ($v_{r.*.2}$) *val* ($v_{r.*.3}$) *where* c_{nd}

then true else und). The objects *var* ($v_{r.*}$), *seq* ($v_{r.s.*}$), *und* ($v_{r.*.1}$), *abn* ($v_{r.*.2}$), *val* ($v_{r.*.3}$) and *where* c_{nd} in this form can be omitted. The omitted objects correspond to *var* (), *seq* (), *und* (), *abn* (), *val* () and *where true*, respectively.

15.10. Configurations operations

The element $conf :: cur$ specifying the current configuration is defined as follows:
$(definition\ \ conf :: cur\ \ then\ \ conf :: cur :: atm)$;
$(interpretation\ \ conf :: cur :: atm\ \ then\ \ f_n)$,
where $[f_n\ s_b] = c_{nf}$.

16. Justification of requirements for conceptual configuration systems

In this section, we establish that CCSs meet the requirements stated in section 1:

1. *The formalism must model the conceptual structure of states and state objects of the IQS.* The conceptual structure of states of the IQS is modelled by elements (attributes, concepts, individuals) and, in more detail, usual and generic conceptuals of conceptual configurations.
2. *The formalism must model the content of the conceptual structure.* The content of the conceptual structure is modelled by conceptual configurations.
3. *The formalism must model information queries, information query objects, answers and answer objects of the IQS.* Information queries, information query objects, answers and answer objects of the IQS are modelled by elements of the CCS.
4. *The formalism must model the interpretation function of the IQS.* The interpretation function of the IQS is modelled by the interpretation function *value* of the CCS.
5. *The formalism must be quite universal to model typical ontological elements.* Models of typical ontological elements is presented in sections 6-10, 12 and 13.
6. *The formalism must provide a quite complete classification of ontological elements, including the determination of their new kinds and subkinds with arbitrary conceptual granularity.* Classification of ontological elements based on the two-level scheme is presented in section 11. The arbitrary conceptual granularity is provided by conceptuals.
7. *The model of the interpretation function must be extensible.* The model of the interpretation function of the IQS is extended by addition of element definitions.

8. *The formalism must have language support. The language associated with the formalism must define syntactic representations of models of states, state objects, queries, query objects, answers and answer objects and includes the set of predefined basic query models.* The CCSL language associated with CCSs defines syntactic representations of models of states, state objects, queries, query objects, answers and answer objects and includes the set of predefined basic query models.

Thus, the requirements are met for CCSs.

17. Comparison of conceptual configuration systems with abstract state machines

Abstract state machines (ASMs) [3, 4] are the special kind of transition systems in which states are algebraic systems. They are a formalism for abstract unified modelling of computer systems. We compare CCSs with ASMs, based on the requirements stated in section 1:

1. *The formalism models the conceptual structure of states of the IQS.* The conceptual structure of states of the IQS is modelled by the appropriate choice of symbols of the signature of an algebraic system. Thus, both ASMs and CCSs model the conceptual structure of states of the IQS, but CCSs make it by more natural ontological way.
2. *The formalism models the content of the conceptual structure.* The content of the conceptual structure is modelled by the interpretation of signature symbols in a particular state.
3. *The formalism must model information queries, information query objects, answers and answer objects of the IQS.* Information queries and information query objects of the IQS are modelled by terms, and answers and answer objects of the IQS are modelled by values of the terms. The element-based representation in CCSs is reacher than the term-based representation in ASMs.
4. *The formalism must model the interpretation function of the IQS.* The interpretation function of the IQS are modelled by the term interpretation function that is simpler than the element interpretation function in CCSs.
5. *The formalism is quite universal to model typical ontological elements.* In contrast to CCSs, typical ontological elements are not naturally modelled by ASMs.
6. *The formalism provides a quite complete classification of ontological elements, including the determination of their new kinds and subkinds with arbitrary conceptual granularity.*

In contrast to CCSs, ASMs do not allow to classify naturally ontological elements and define their new kinds and subkinds with arbitrary conceptual granularity.

7. *The model of the interpretation function must be extensible.* The model of the interpretation function can not be directly extended in ASMs.
8. *The formalism must have language support.* There are two languages AsmL [5] and XasM [6] for specification of ASMs. The AsmL language is more expressive than CTSL. It is fully integrated into the Microsoft .NET environment and uses XML and Word for literate specifications. XASM realizes a component-based modularization concept based on the notion of external functions as defined in ASMs.

18. Conclusion

In the paper two formalisms (information query systems and conceptual configuration systems) for abstract unified modelling of the artifacts of the conceptual design of closed information systems have been proposed. The basic definitions of the theory of CCSs have been given. The classification and interpretation of elements of such conceptual structures of CCSs as conceptuals, conceptual states, conceptual configurations, concepts and attributes has been presented. The classification of ontological elements based on these conceptual structures has been described. A language of CCSs has been defined.

The feature of conceptual design for closed information systems based on conceptual configuration systems is that they allow us to describe the conceptual structure of states of the information systems in detail. We plan to extend this formalism to describe both states and state transitions in detail and apply it for conceptual design of wider class of information systems.

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